# ADVANCED STEERING STABILITY CONTROLS FOR ARTICULATED VEHICLES BY USING BOND GRAPHS AND MODEL PREDICTIVE CONTROL

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**ABSTRACT**

The globalization of trade, the economy and employment have made the transport industry one of the main sectors where development is evident and constant. In particular, it is worth noting the evolution obtained in the transport of goods. This is clearly differentiated into four main types of transport: rail, air, sea and land. The latter remains, despite the years, the most used by companies especially for domestic trade due to its great flexibility.

The above-mentioned land or road transport is mainly carried out from compound vehicles, that is, a tractor unit articulated to an extensive set of trailers and semi-trailers depending on the merchandise to be transported. However, despite the improvements carried out on most of the main roads nationwide and the experience of drivers who are immersed in this sector, driving this type of vehicle continues to be complex. Although the number of accidents that occur on the road for trucks and articulated vehicles is small compared to those produced with passenger cars, it is still very high. This number is also considerably higher than in other freight transport.

Some of these aspects allow us to understand the difficulty that occurs when getting behind the wheel of these vehicles. However, as also happens with passenger cars, driving assistance systems (lane control, adaptive speed control, etc.) allow reducing on certain occasions what could be an insured accident.

Based on the latter, the fundamental motivation of this paper has been to provide "intelligence" to the articulated vehicles in order to convert it into an autonomous vehicle. This can be obtained from the development of a control system based on predictive model control (MPC). From this control system it is intended that the vehicle by itself can carry out the usual driving maneuvers, achieving the adaptation of the vehicle to a previously fixed path. In addition, it is intended to introduce a stability control to the articulated vehicle to avoid its instability under high speeds and forced trajectories. This steering stability control will be based on differential braking.

**Keywords:** Autonomous vehicle, Articulated truck, Differential braking, Model Predictive Control, Bond Graphs

## 1 INTRODUCTION

Nowadays the road freight transportation dominates the freight transportation sector being significantly higher than the second method of transport (the maritime transport). For this reason, it's necessary to develop new technologies such as autonomous vehicles to reduce the risks generated by this sector and to be able to satisfy the more demanding legislation.

The land or road transport is mainly carried out from compound vehicles, that is, a tractor unit articulated to an extensive set of trailers and semi-trailers depending on the merchandise to be transported. However, despite the improvements carried out on most of the main roads nationwide and the experience of drivers who are immersed in this sector, driving this type of vehicle continues to be complex. Although the number of

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accidents that occur on the road for trucks and articulated vehicles is small compared to those produced with passenger cars, it is still very high. This number is also considerably higher than in other freight transport.

The snaking phenomenon can occur with any car/trailer combination. Snaking normally occurs at high speeds. But there is a risk of it occurring at lower speeds if the trailer is overloaded or the load is improperly distributed, e.g., too far back.

If snaking has started, it could be difficult or even impossible to suppress. This makes the car/trailer combination difficult to control and there is a risk that you could, for example, end up in the wrong lane or leave the carriageway. Figure 1 shows an example of this instabilities.

Figure 1. The snaking phenomenon that can occur with any car/trailer combination

In order to avoid these situations, especially critical in an autonomous truck, but also in a driven truck, this paper proposes a stability control that prevents these situations and remains the truck stable.

The trailer stability controller function continually monitors the car's movements, particularly lateral movements, and serves to stabilize the car/trailer combination. In this paper we present the design of a control system for yaw stability for an autonomous driving semi-trailer truck based on Model Predictive Control (MPC).

Another research challenge in the automotive industry are autonomous vehicles, that are defined as the vehicles that can recognize its surroundings and imitate the human ability of driving and control [1]. The autonomous capacity that a vehicle has can be measured in 6 levels. Nowadays, different companies are working in the development of level 4 and 5 vehicles [2], even though the highest level that can be found in the market is the Tesla Autopilot (level 3).

With respect to the control system, the MPC is an advanced control method [3]. It was used for the first time in the beginning of the 70s by Shell Oil. This technique has been successfully used in many industrial applications, like thermal energy control [4], collision avoidance [5], vehicle stability [6], and energy management [7]. The MPC capacity of working with non-linear systems makes it appropriate for several applications in engineering, being of particular interest its use in autonomous vehicles [8].

In the MPC, the model is used to predict the changes in the states of the system produced by the input variables. The system uses the plant measures, the current dynamic states, the limits imposed by the user in the variables and the objective variables to calculate the future changes in the state variables. Thanks to these changes, the state variables can be close to the objective value while satisfying the imposed conditions.

Some companies in the automotive industry like Ford, BMW, Honda, PSA or Toyota are already studying the implementation of this kind of control systems. Its application covers the traction control, the semi active suspension control, the stability control or the management of the energy in electric vehicles. Even though this type of control system requires a high computational cost, the combination of multi parametric programming and an appropriate design of the prediction horizons can reduce to a suitable time the computational effort for the vehicle microcontrollers, obtaining real time in the real on-board systems. MPC theory and applications to autonomous vehicles can be seen in [9], [10], [11] and [12].

The main contribution of this paper is to provide “intelligence” to the articulated vehicles in order to convert it into an autonomous vehicle. This can be obtained from the development of a control system based on predictive model control (MPC). From this control system it is intended that the vehicle by itself can carry out the usual driving maneuvers, achieving the adaptation of the vehicle to a previously fixed path. In addition, it is intended to introduce a stability control to the articulated vehicle to avoid its instability under high speeds and forced trajectories. This steering stability control will be based on differential braking. Differential braking is the unequal application of braking applied to different wheels. Globally it produces a torque that acts as a yaw stability control mode.

The remainder of this paper is organized as follows. Section II describes the theoretical background of stability for articulated vehicles. Section 3 describes the model developed for the trailer. Section describes 4 the control model. This controller is based in MPC, which is appropriate for this task due to its considerable advantages and its capability to control multivariable systems time-varying being a robust method. Section 5 presents the results of several simulations, and Section 6 includes the conclusions.

## 2 THEORETICAL BACKGROUND.

The stability of single vehicles has thoroughly been investigated, amongst others by Pacejka [13], Besselink [14], and Pauwelussen [15]. A lot of work has been performed, both numerically and analytically, to quantify stability boundaries.

Pacejka [13], Andrzejewski et al. [4], Ellis [17], Pauwelussen [15] et al., Hac et al. [18], and Troger et al. [19], studied the stability of vehicles with one articulation, such as tractor-semitrailers, truckcentre axle trailers and car-caravan combinations. Numerical analyses were used by Pacejka and Andrzejewski to find the bifurcations and investigate the effect of parameter changes on stability.

Ellis and Pauwelussen derived an algebraic equation for the stability boundary as function of the vehicle parameters.

In general, the following types of instability may occur [13], [19], [17]:

1. Divergent instability. The mass of the trailer is partly supported by the towing vehicle in the coupling point. The vehicle combination makes a monotonically unstable motion when the vertical force of the trailer on the towing vehicle becomes too large.

2. An unstable oscillatory motion. The snaking oscillation amplitude is unlimited, also in the non-linear case. The slip angle increases with increasing amplitudes which lowers the average cornering stiffness because of the digressive non-linear tire cornering force characteristic. This will make the situation increasingly worse.

Trailer swing is also often seen as a yaw instability, although the combination may be stable in a mathematical sense. The following two types of trailer swing can be distinguished [17]. They are illustrated in figure 2.

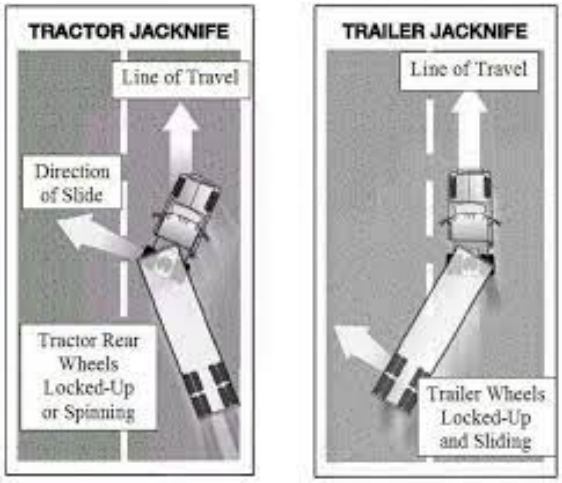



Figure 2: Two jack-knife possibilities of a tractor-semitrailer

In figure 2 we can see as the tractor may jack-knife either under power or when braking, when the rear wheels of the tractor loose grip or traction. Also, we can see as the trailer swing may occur when the trailer axle wheels are locked during braking. A very complete analysis of articulated vehicles can be found in [20].

## 3 DYNAMIC MODEL

### *3.1 Vehicle model*

The vehicle model is composed by two bodies, tractor and semitrailer, connected with an ideal revolute joint in the kingpin. The vehicle has two axles in the tractor and three axles in the semi-trailer and has been simplified by grouping the three semi-trailer axles in only one equivalent. Each axle has its equivalent left and right wheels. The truck has the driving system in the tractor rear axle, meanwhile the steering is applied in the front wheels.

With these assumptions, the vehicle model consists of a 2D model with 4 degrees of freedom, corresponding to the longitudinal and lateral displacements of the tractor, the tractor yaw angle and the semi-trailer yaw angle.

Fig. 3 shows the vehicle schematics and the parameters considered in the model. The values of the parameters used in the model are relegated to Appendix I.

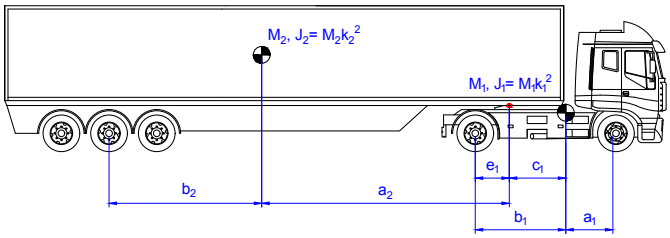



Figure 3. Model vehicle parameters

A global inertial reference frame $[X \quad Y]$ and two local reference frames $[x_1 \quad y_1]$ and $[x_2 \quad y_2]$ are defined. The tractor is represented as the body 1 and uses subscript 1 and the semitrailer is represented with subscript 2. Figure 4 shows the model variables.

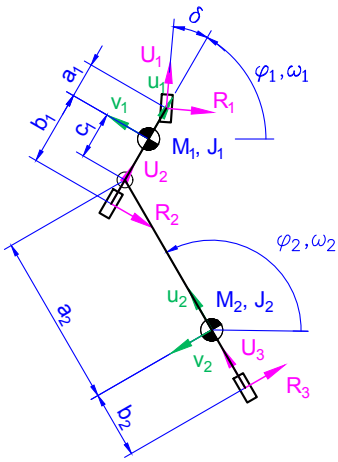



Figure 4. System variables definition

Capital letters represent variables referred to the global reference frame, small letters represent variables referred to the local reference frame, and greek letters represent rotational variables. The notation is defined as follows:

$[X_i \quad Y_i]$ with $i = 1, 2$ are the c.o.g. (center of gravity) positions referred to the inertial reference frame $[X \quad Y]$ for each body.

$[u_i \quad v_i]$ with $i = 1, 2$ are the c.o.g. velocities referred to the local reference frame.

$\varphi_i$ with $i = 1, 2$ are the orientation of the local reference frame with respect to the inertial reference frame.

$\varphi_{ij}$ means $\varphi_i - \varphi_j$ $s\varphi_i$ means $sin\varphi_i$ $c\varphi_i$ means $cos\varphi_i$

$s\varphi_{ij}$ means $sin(\varphi_i - \varphi_j)$ $c\varphi_{ij}$ means $cos(\varphi_i - \varphi_j)$

$\omega_i$ with $i = 1, 2$ are the tractor and semitrailer yaw velocities. $\omega_{ij}$ means $\omega_i - \omega_j$

$X = [X_1 \quad X_1 \quad \varphi_1 \quad X_2 \quad Y_2 \quad \varphi_2]^T$ is the vector of coordinates that represent the system movement in the inertial frame.

$v = [u_1 \quad v_1 \quad \omega_1 \quad u_2 \quad v_2 \quad \omega_2]^T$ is the vector of the c.o.g. velocities referred to the local frames.

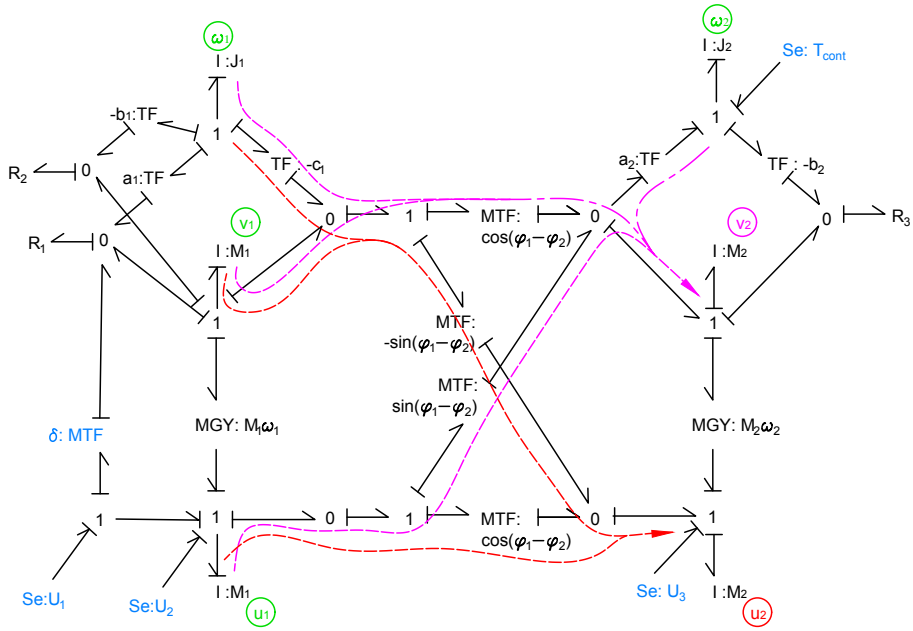



Figure 5. Bond graph representing the tractor-semitrailer model

### *3.2 Bond graph model*

With these assumptions, and following the guidelines shown in [21], we can build the following bond graph representing the tractor-semitrailer model (fig. 5).

In this bond graph model, the meaning of the different components is as follows:

$[U_1 \quad U_2 \quad U_3]$ are the driving/braking forces in the front trailer axle, rear trailer axle and semitrailer axle. All of them are braking forces when the vehicle brakes, but, when the truck is acceleration, only $U_2$ works.

$[R_1 \quad R_2 \quad R_3]$ represent the lateral tire forces in the front trailer axle, rear trailer axle and semitrailer axle.

The MTFs structure between body 1 and body 2 is used to model the articulation in the kingpin.

$[\delta]$ represents the tractor steering angle

$[T_{cont}]$ represents the stability controller, introduced as differential braking, explained in the next section

The bond graph presented in Figure 5 shows that, assigning causality, there are four degrees of freedom $V_I = [u_1 \quad v_1 \quad \omega_1 \quad \omega_2]$ and two dependent velocities $V_D = [u_2 \quad v_2]$. Then, we can obtain the following expressions for the dependent velocities, written in terms of the independent ones.

$$\begin{aligned} u_2 &= u_1\, c\varphi_{12} - v_1\, s\varphi_{12} + c_1\, \omega_1\, s\varphi_{12} \\ v_2 &= u_1\, s\varphi_{12} + v_1\, c\varphi_{12} - c_1\, \omega_1\, c\varphi_{12} - a_2\omega_2 \end{aligned} \tag{1}$$

And, considering that we are considering local reference frames, we can obtain the accelerations as follows from the bond graph:

$$\begin{aligned} {du_1}/{dt} &= \dot{u}_1 - \omega_1 v_1 \\ {dv_1}/{dt} &= \dot{v}_1 + \omega_1 u_1 \\ {du_2}/{dt} &= \dot{u}_2 - \omega_2 v_2 \\ {dv_2}/{dt} &= \dot{v}_2 + \omega_2 u_2 \end{aligned} \tag{2}$$

Solving the effort assignments, and writing the dependent accelerations in terms of the independent ones by using (1) and (2), we obtain the following state equations:

$$\begin{bmatrix} M_1 + M_2 & 0 & 0 & -M_2 a_2\, s\varphi_{12} \\ 0 & M_1 + M_2 & -M_2 c_1 & -M_2 a_2\, c\varphi_{12} \\ 0 & -M_2 c_1 & J_1 + M_2 {c_1}^2 & M_2 a_2 c_1\, c\varphi_{12} \\ -M_2 a_2 s\varphi_{12} & -M_2 a_2 c\varphi_{12} & M_2 a_2 c_1 c\varphi_{12} & J_2 + M_2 {a_2}^2 \end{bmatrix} \begin{bmatrix} \dot{u}_1 \\ \dot{v}_2 \\ \dot{\omega}_1 \\ \dot{\omega}_2 \end{bmatrix} =$$

$$\begin{bmatrix} 0 & M_1\omega_1 + M_2(\omega_1 + \omega_{12}) & -M_2 c_1 \omega_{12} & 0 \\ -M_1\omega_1 - M_2(\omega_1 + \omega_{12}) & 0 & 0 & 0 \\ M_2 c_1 \omega_1(\omega_1 + \omega_{12}) & 0 & 0 & 0 \\ M_2 a_2 c\varphi_{12}(\omega_1 + \omega_{12}) & -M_2 a_2\, s\varphi_{12}(\omega_1 + \omega_{12}) & M_2 a_2 c_1 \omega_{12}\, s\varphi_{12} & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ \omega_1 \\ \omega_2 \end{bmatrix} + \tag{3}$$

$$\begin{bmatrix} U_1 + U_2 + U_3\, c\varphi_{12} - \delta R_1 + R_3\, s\varphi_{12} \\ -U_3\, s\varphi_{12} + R_1 + R_1 + R_3\, c\varphi_{12} \\ U_3 c_1\, s\varphi_{12} + a_1 R_1 - b_1 R_2 - c_1 R_3\, c\varphi_{12} \\ -(a_2 + b_2) R_3 \end{bmatrix}$$

To obtain the displacements in the global reference frame, we use the following equations:

$$\begin{bmatrix} \dot{X}_1 \\ \dot{Y}_1 \\ \dot{\varphi}_1 \\ \dot{\varphi}_2 \end{bmatrix} = \begin{bmatrix} c\varphi_1 & -s\varphi_1 & 0 & 0 \\ s\varphi_1 & c\varphi_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ \omega_1 \\ \omega_2 \end{bmatrix} \tag{4}$$

### *3.3 Tire forces*

Lateral tire forces, $R_i$, are required to negotiate a curve. They result in tire slip angles. This tire characteristic is linear for small slip angles. The gradient of the curve in the linear regime is the tire cornering stiffness, $C_i$

A normalized cornering stiffness is used to study the effect of scaling the tire cornering stiffness linearly with vertical load on dynamic stability. It was found in

[14] that, in contrast to passenger car tires, the relation between the tire cornering stiffness and vertical load is nearly linear for truck tires. It was also found in [14] that the characteristic shows an even more linear relationship if dual tires are applied, which is often the case in truck configurations, except for the steered axle.

This means that the assumption that the cornering stiffness versus load characteristic is in its linear region is often true for truck tires.

The cornering stiffnesses are calculated as function of vertical load, with $f = f_1 = f_2 = f_3 = 5.73\ [1/\text{rad}]$ using expressions in (5) and (6)

$$\begin{aligned} C_1 &= f\,F_{Z1} \quad F_{Z1} = M_1 g \frac{b_1}{l_1} - M_2 g \frac{b_2}{l_2}\frac{e_1}{l_1} \\ C_2 &= f\,F_{Z2} \quad F_{Z2} = M_1 g \frac{a_1}{l_1} + M_2 g \frac{b_2}{l_2}\frac{l_{1s}}{l_1} \\ C_3 &= f\,F_{Z3} \quad F_{Z3} = M_2 g \frac{a_2}{l_2} \end{aligned} \tag{5}$$

with:

$$\begin{aligned} l_1 &= a_1 + b_1 \\ l_2 &= a_2 + b_2 \\ l_{1s} &= a_1 + b_1 + e_1 \end{aligned} \tag{6}$$

The lateral tire forces $R_i$ are obtained from the slip angle $\alpha_i$, that depends on the relation between the lateral and longitudinal wheel speed on the ground contact point, according to the following expressions (7) by multiplying the slip angle by the cornering stiffness $C_i$:

$$\begin{aligned} R_1 &= C_1\alpha_1 \quad \alpha_1 = (v_1 + a_1\omega_1)/u_1 - \delta \\ R_2 &= C_2\alpha_2 \quad \alpha_2 = (v_1 - b_1\omega_1)/u_1 \\ R_3 &= C_3\alpha_3 \quad \alpha_3 = (v_2 - b_2\omega_2)/u_2 \end{aligned} \tag{7}$$

The model assumes small steering angles, so $\sin\delta \cong \delta$ and $\cos\delta \cong 1$. The maximum steering angle is defined in the model constraints (14) - (15) and in Appendix I.

## 4 DESIGN OF THE MPC CONTROLLER

### *4.1 Problem description*

In this section we are defining an optimization problem in order to define the controllers that we need to guide the vehicle and to keep yaw stability. We stablish and optimal control design starting from the system dynamics of the trailer. We consider as inputs to be controlled the driving/braking force at each axle, the steering angle, and the torque that is used to model the differential braking used in the yaw stability control. Then, we stablish a set of operational constraints by fixing the limits that we allow to the inputs and to the outputs. We also define an objective function that allows to stablish the goals of the control problem: tracking a desired trajectory and keep yaw stability.

### *4.2 Optimal control design*

Regrouping the previous expressions (3) and (4) the system equations for the vehicle dynamics are written in a compacted way as:

$$\dot{x} = f_v(x, U) \tag{8}$$

where:

$x = [V_I{}^T \quad X_i{}^T]^T$ is a vector including the states.

$V_I = [u_1 \quad v_1 \quad \omega_1 \quad \omega_2]^T$ is vector with the independent velocities.

$X_i = [X_1 \quad Y_1 \quad \varphi_1 \quad \varphi_2]^T$ includes the global positions and orientations for tractor and semitrailer.

$U = [U_1 \quad U_2 \quad U_3 \quad \delta \quad T_{cont}]^T$ is a vector including the inputs.

A Model Predictive Control (MPC) approach is proposed for the vehicle. MPC is selected due to its capability of systematically handling multiple input and state constraints, which in this problem are critical. According to the receding horizon principle, at each time step the MPC algorithm computes the optimal control and state trajectories solving a finite horizon optimization problem.

For the formulation of the MPC a prediction horizon $[t, t + N_p]$ is considered at time *t*. The notation $x_{t+k|t}$ represents the state vector at time $t + k$, predicted at time $t$, obtained by starting from the current state $x_{t|t} = x(t) \equiv x_t$, and where $U_{\cdot|t} = [U_{t|t}, \dots, U_{t+N_p-1|t}]$ denotes the unknown input variables to be optimized. As previously stated, the subscript 1 denotes the tractor and subscript 2 denotes the semitrailer.

#### *4.2.1 System dynamics*

Equations (8) represents the system dynamics updates for the discrete-time model obtained from (3) and (4). The initial state is set in (9)

$$\begin{aligned} &x_{k+1|t} = f(x_{k|t}, U_{k|t}) \\ &\forall k = t, \dots, t + N_p - 1 \end{aligned} \tag{8}$$

$$x_{t|t} = x_t \tag{9}$$

### *4.2.2 Model constraints*

#### 4.2.2.1 Input constraints

We consider as inputs to be controlled the driving/braking force at each axle, the steering angle, and the torque that is used to model the differential braking used in the yaw stability control.

The driving and braking forces must be bounded by:

$$-F_{b1} \leq U_{1_{k|t}} \leq 0 \quad (10)$$

$$-F_{b2} \leq U_{2_{k|t}} \leq F_{d2} \quad (11)$$

$$-F_{b3} \leq U_{3_{k|t}} \leq 0 \quad (12)$$

$$-P_b \leq u_{1_{k|t}}\left(U_{1_{k|t}} + U_{2_{k|t}}\right) + u_{2_{k|t}} U_{3_{k|t}} \leq P_d \quad (13)$$

$$\forall k = t, \dots, t + N_p - 1$$

The driving / braking forces constraints include the limitation of the maximum driving and braking force and the power limitation in driving and braking. Eq. (10) to (13) represent the maximum driving/braking forces and the maximum power for driving/braking. Eq. (10) and (12) are bounded between $[-F_{bi}, 0]$because axles 1 and 3 only break, instead of axel 2 (11), bounded between $[-F_{b2},\ F_{d2}]$ because in this axle, we have de traction.

Constraints (14) and (15) bounds the steering angle and the steering rate.

$$-\delta_{max} \leq \delta_{k|t} \leq \delta_{max} \quad (14)$$

$$-\dot{\delta}_{max} \leq (\delta_{k|t} - \delta_{k-1|t})\Delta t/ \leq \dot{\delta}_{max} \quad (15)$$

$$\forall k = t, \dots, t + N_p - 1$$

And constraints (16) constraints the differential braking stability controller

$$-T_{max} \leq T_{cont} \leq T_{max} \quad (16)$$

In the MPC formulation, we will refer to all these constraints as $u_{k|t} \in \mathbb{U}_{k|t}$.

#### 4.2.2.2 Speed constraint

In order to guarantee that the vehicle respects the speed limit, the speed is bounded by the following constraint:

$$v_{min} \leq u_{1_{k|t}} \leq v_{max}$$
$$\forall k = t, \dots, t + N_p - 1 \quad (17)$$

In the MPC, we will refer to this constraint as $v_{k|t} \in \mathbb{V}_{k|t}$.

### *4.2.3 Cost function*

The objective of the truck is to circulate at the speed fixed by a reference, following a trajectory previously define, and preserving yaw stability. The objective control may be stablished with an objective function composed by different terms. This objective function is used to minimize the different errors produced in the trajectory tracking and in the relative yaw rate, as shown in figure 6.

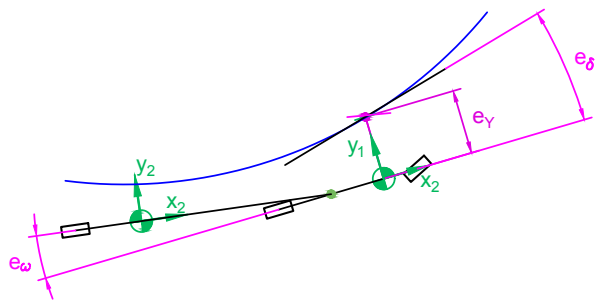



Figure 6. Errors to be minimized in the cost function

Therefore, the objective function $J(x, U)$ (19) is defined as follows:

$$J(x, U) = \sum_{k=t}^{t+N_p} K_V \left(u_{1_{k|t}} - u_{ref}\right)^2 + \quad (18)$$

$$+ \sum_{k=t}^{t+N_p} K_Y\ e_Y{}^2 \quad (19)$$

$$+ \sum_{k=t}^{t+N_p} K_\alpha\ e_\delta{}^2 \quad (20)$$

$$+ \sum_{k=t}^{t+N_p} K_{cont} e_\omega{}^2 \quad (21)$$

with

$$e_Y = y_{1_{k|t}} - Y_{k|t} \quad (22)$$

$$e_\delta = \varphi_{1_{k|t}} - \alpha_{k|t} \quad (23)$$

$$e_\omega = \dot{\varphi}_{1_{k|t}} - \dot{\varphi}_{2_{k|t}} \quad (24)$$

The different terms in the cost function (19) to (21) have the following meaning: $K_V \geq 0$ represents the weight penalizing the output deviation from the truck maximum desired speed, being $u_{ref}$ the maximum desired speed. $K_Y \geq 0$ represents the weight penalizing

the lateral displacement in the trajectory tracking, being $Y_{k|t}$ the Y coordinate of the desired trajectory. $K_\alpha \geq 0$ represents the weight penalizing the orientation in the trajectory tracking being $\alpha_{k|t}$ the orientation of the desired trajectory.

The term (21) is the key point in the yaw stability control. This term minimizes the yaw rate between tractor and semitrailer, so it preserves the yaw instability.

#### *4.2.4 Model predictive control formulation*

As previously said, the objective of the truck is to circulate at the speed fixed by a reference, following a trajectory previously define, and preserving yaw stability.

Therefore, the optimization problem is formulated as:

$$\min_{U_{\cdot|t}} \; J(x, U) \tag{25}$$

subject to:

$$x_{k+1|t} = f(x_{k|t}, U_{k|t}) \tag{26}$$

$$U_{k|t} \in \mathbb{U}_{k|t} \tag{27}$$

$$v_{k|t} \in \mathbb{V}_{k|t} \tag{28}$$

$$\forall k = t, \dots, t + N_p - 1$$

$$x_{t|t} = x_t \tag{29}$$

The resulting optimal states and inputs of (25) – (29) are denoted as following:

$$\begin{aligned} x_t^* &= \begin{pmatrix} x_{t|t}^* & x_{t+1|t}^* & \cdots & x_{t+N_p|\mathrm{t}}^* \end{pmatrix}^T \\ U_t^* &= \begin{pmatrix} U_{t|t}^* & U_{t+1|t}^* & \cdots & U_{t+N_p|\mathrm{t}}^* \end{pmatrix}^T \end{aligned} \tag{30}$$

For closing the loop, the first input is applied to the system (8) during the time interval $[t, t+1)$

$$U_t = U_{t|t}^* \tag{31}$$

At the next time step *t+1*, a new optimal problem in the form of (25) – (29) is solved over a shifted horizon, based on a new states measurement.

## 5 SIMULATIONS

Two different models have been developed for the design and validation through simulation of the yaw stability controller. The first one corresponds to a trailer without yaw stability controller and the second one includes the stability controller. So, in the first model, the term (21) of the cost function is not considered, while in the second model in which we introduce this term.

We have defined a velocity profile following a sinusoidal line, corresponding to the following equation (32):

$$\begin{aligned} Y_{road}(\mathrm{X}) &= 5 * \left(1 - \cos\left(\frac{\pi}{200} X\right)\right) \\ \alpha_{road}(\mathrm{X}) &= \mathrm{atan}\left(5 * \frac{\pi}{200} \sin\left(\frac{\pi}{200} X\right)\right) \end{aligned} \tag{31}$$

For the trailer parameters included in Annex I, following the expression obtained in [20], we obtain a critical speed of 31,04 m/s. Then, we have checked the stability control in two cases. The first one is with a reference speed of 15m/s, which is below the critical speed. In this case, the trailer is stable without stability controller. We can see this in figures 7, 8 and 9

Figure 7 shows the trailer behavior without stability controller. We can see in this figure that the trailer remains stable and tracks the desired trajectory. Figure 8 shows the trailer behavior with the stability controller. If we compare these two figures, we can see how, although in the case without a controller the vehicle remains stable, the relative yaw angle between tractor and semitrailer is slightly higher than in the case with a controller.

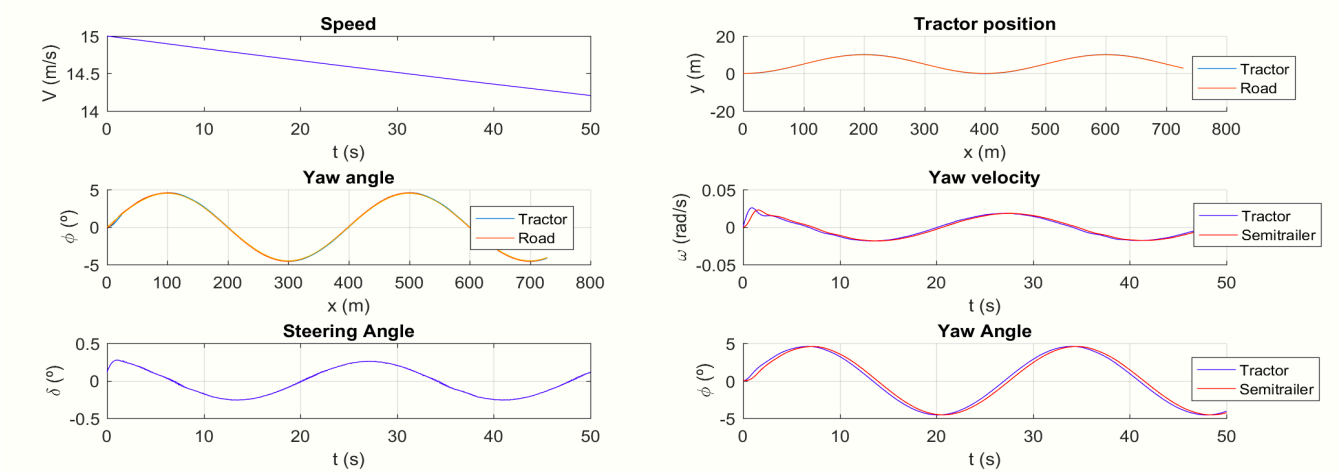



Figure 7. Trailer behavior without stability controller for $v_{ref}$= 15m/s.

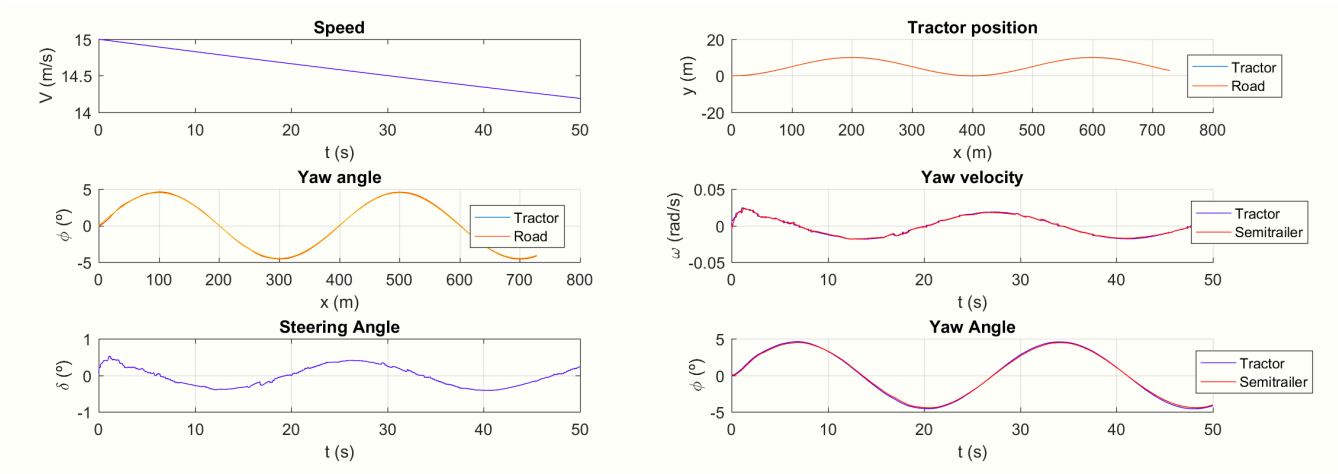
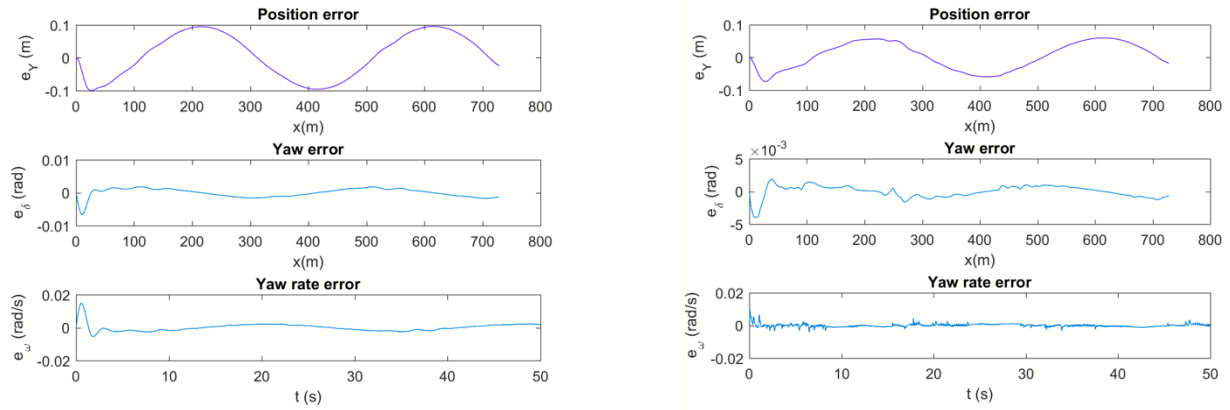



Figure 8. Trailer behavior with stability controller for $v_{ref}$= 15m/s.

Figure 9 shows the tracking errors. We can see that the lateral position and yaw errors are enough small, and the truck follows the desired trajectory. And we can also observe that the yaw rate error is also enough small, and the trailer remains stable in both cases (without and with yaw stability control).




(a) Without yaw stability control (b) With yaw stability control

Figure 9. Tracking errors for $v_{ref}$= 15m/s

The second simulation case considers a reference speed of 35m/s, which is greater than the critical speed. In this case, the trailer is unstable without stability controller. We can see this in figures 10, 11 and 12. We can see in figure 10 that the trailer becomes unstable and do not track the desired trajectory The trailer swings and after that, it reduced its speed trying to attain a velocity in which it remains stable. Figure 11 shows the trailer behavior with the stability controller. We can see now how the trailer remains stable.

Figure 12 shows the tracking errors. In this figure we can observe the unstable behavior of the trailer without stability control compared with the trailer with stability control.

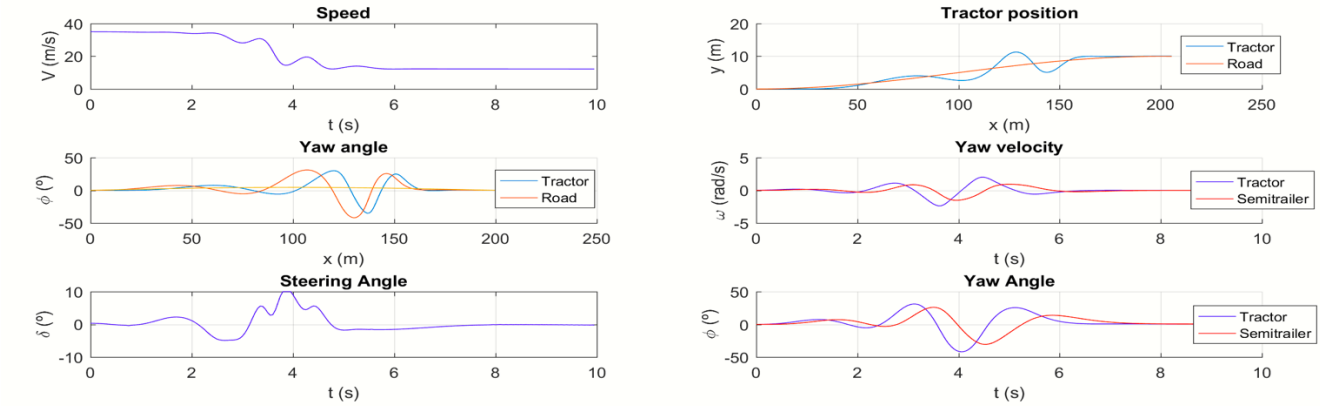



Figure 10. Trailer behavior without stability controller for $v_{ref}$= 35m/s.

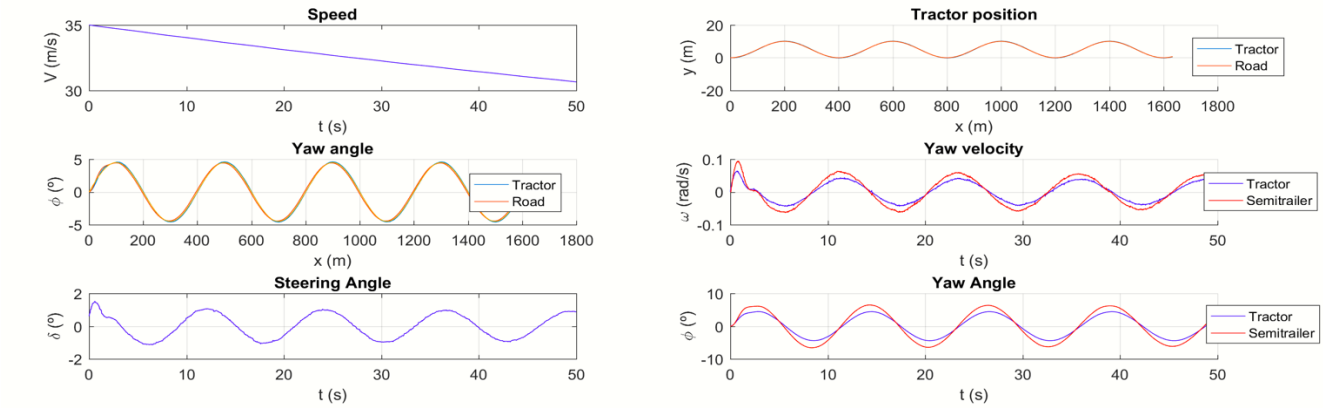
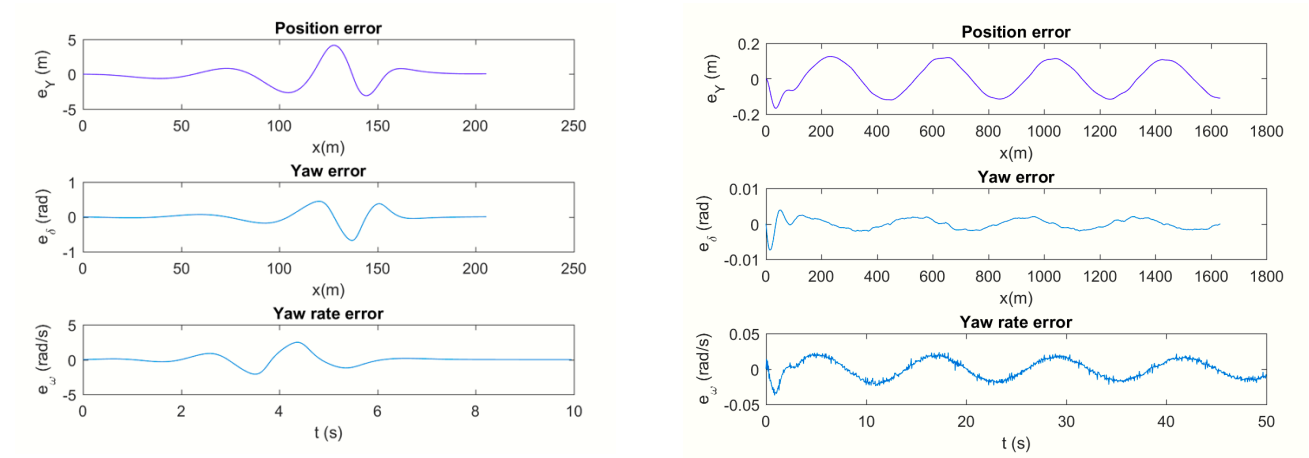


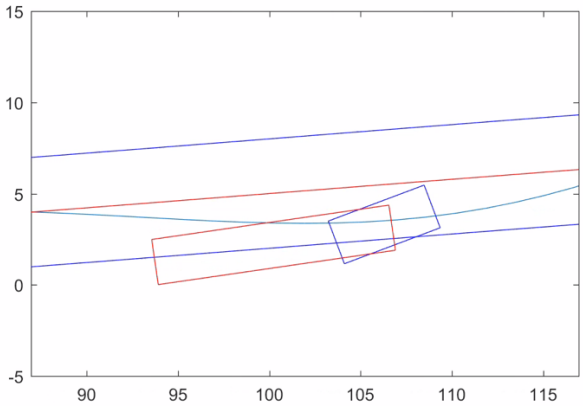

Figure 11. Trailer behavior with stability controller for $v_{ref}$= 35m/s.



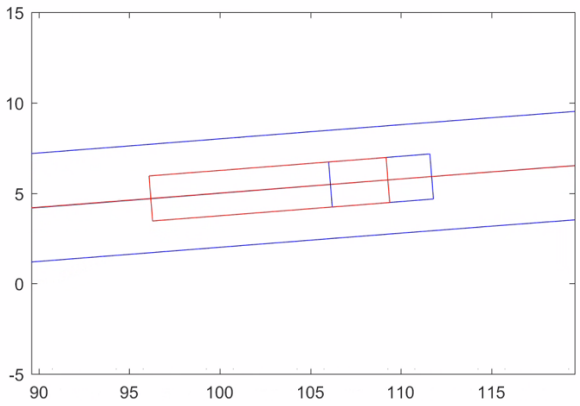

(a) Without yaw stability control (b) With yaw stability control

Figure 12. Tracking errors for $v_{ref}$= 35m/s

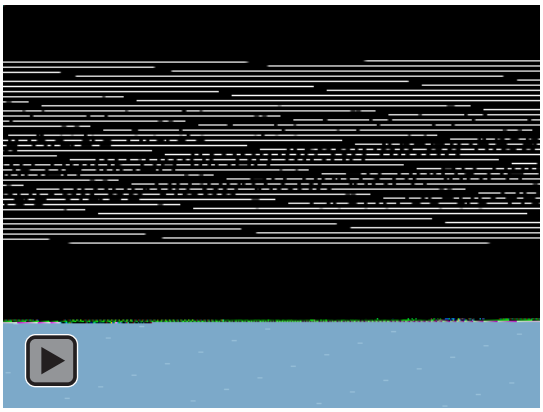
In the following pictures and videos, we can observe the trailer movement when the trailer is stable and when is unstable.



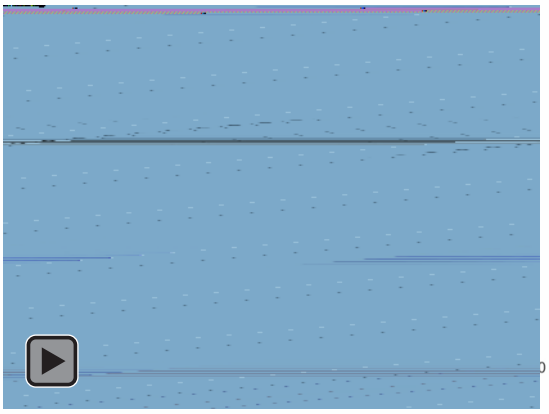

(a) Without yaw stability control




(b) With yaw stability control



(c) Video without yaw stability control



(d) Video with yaw stability control

Figure 13. Picturesdifferen and Videos of the trailer movement for $v_{ref}$= 35m/s

## 6 CONCLUSIONS

In this paper, a simulation model has been developed, representing faithfully the expected behavior of an autonomous driving vehicle constituted by a trailer composition.

The main contribution of this paper has been to provide "intelligence" to the articulated vehicles in order to convert it into an autonomous vehicle. From this control system it is intended that the vehicle by itself can carry out the usual driving maneuvers, achieving the adaptation of the vehicle to a previously fixed path. In addition, it is intended to introduce a stability control to the articulated vehicle to avoid its instability under high speeds and forced trajectories. This steering stability control will be based on differential braking.

In the first part of the paper, the articulated vehicle model has been presented including the mechanical behavior.

Next, a control system based on MPC has been implemented. The MPC ability to control multivariable systems that are also time-varying, defining constraints to the vehicle states is especially useful for this application, stablishing limitations for guarantying the lateral and yaw stability of the trailer.

Once the controller has been implemented, different simulations have been developed in which the trailer carried out different maneuvers autonomously. These

maneuvers show the behavior of the yaw stability controller.

The simulation results have been satisfactory obtaining interesting results that show how with this stability control we can avoid the instability problems that appear in articulated trucks.

The behavior presented by the model during the simulations was similar to the expected, performing all maneuvers autonomously in conditions of stability and safety. However, this paper includes only simulation results. The validations including the comparative results between the model and the real vehicle will be object of future research, building, for example, a scaled prototype or introducing sensors in a real vehicle and making itineraries in a test circuit that will be simulated later with the model, comparing both.

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## APPENDIX I: VEHICLE PARAMETERS

| Parameter | Description | Value |
|---|---|---|
| $M_1$ | Tractor mass | 7449 Kg |
| $k_1$ | Radius of gyration | 1.89 m |
| $M_2$ | Semi-trailer mass | 32551 Kg |
| $k_2$ | Radius of gyration | 4,05 m |
| $a_1$ | Dist. tractor front - c.o.g. | 1.10 m |
| $b_1$ | Dist. tractor rear axle - c.o.g. | 2.49 m |
| $b_2$ | Dist. semitrailer axle - c.o.g. | 3.15 m |
| $a_2$ | Dist. kingpin semitrailer - c.o.g. | 4.98 m |
| $c_1$ | Dist. kingpin tractor dist. - c.o.g. | 1.81 m |
| $f$ | Normalizad cornering stiffness | 5.73 1/rad |
| $P_d$ | Power (drive) | 500 kW |
| $P_b$ | Power (brake) | 700 V |
| $\delta_{max}$ | Max. steering angle | 10º |
| $\dot{\delta}_{max}$ | Max. steering rate | 0,17 rad/s |