# Performance of Multi-Pair Two-Way Full-Duplex Massive MIMO Relay Systems with Direct Link

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***Abstract*—In this paper, we consider a multi-pair two-way full-duplex relaying system with multiple-input-multiple-output (MIMO) users. Each pair of users exchange information with the aid of a massive MIMO amplify-and-forward (AF) relay, and correlation between the antennas both at the users as well as the relay is considered. The direct link between all user nodes is assumed to be non-negligible, which is suitable for practical urban scenarios. The low-complexity transceiver design at the relay based on maximum ratio combining/maximum ratio transmission (MRC/MRT) processing is presented. The performance of the system is evaluated in terms of the achievable sum-spectral efficiency under two communication schemes. The first scheme attempts to make use of the joint benefits of the relayed and direct links, while in the second scheme the direct link is considered as interference. Comparison analysis show that the first scheme outperforms the second in the presence of a strong direct link. Moreover, the detrimental effect of spatial correlation between the antennas at each user node is investigated.**

***Index Terms*—massive MIMO, full-duplex, two-way relaying, linear processing, direct link**

## I. Introduction

Massive MIMO and full-duplex (FD) relaying are new technologies proposed to enhance overall system performance. In FD relaying, the relay transmits and receives simultaneously at the same frequency and time resource. The problem with the FD operation lies in the self-loop interference caused by signal leakage between the relay output and input [1]. In several previous works [2]–[4], it was shown that massive MIMO at the relay can be used to eliminate the effect of self-loop interference due to the FD operation, if the power scaling scheme is properly selected. As a result, FD relaying combined with massive MIMO has the potential for the development of energy and spectral efficient cellular systems [5]. Two types of FD relaying were presented in the literature: one-way and two-way relaying. FD two-way relaying can further improve system capacity by achieving bidirectional data transmission and reception at the user nodes on the same carrier frequency simultaneously [6]. The results in [6] show that FD two-way relaying can achieve higher data rate than one-way relaying in the medium to high signal-to-noise ratio (SNR) region, however, suffers from a certain loss in the outage performance.

In [7], [8], studies of multi-pair FD relaying with massive MIMO have shown that the self-loop interference can be mitigated with a largescale antenna array at the relay using linear low complexity MRC/MRT or zero-forcing (ZF) processing, however, [7] assumed that there is no direct link between user nodes which might not be the case in practice. In [4], the direct link between user nodes is considered with a proposed transmission scheme to capture the joint benefits of both the direct as well as the relayed link, and the achievable sum-rate of the FD two-way relaying system with only single-input-single-output (SISO) users was derived. A model where multiple pairs of MIMO users exchange information via a FD relay with MRC/MRT processing was developed in [9], this time assuming that user nodes are clustered into two groups with each user node in a group, exchanging information with a user node in the other group. Only the direct links between user nodes belonging to the same group were considered, whereas for practical urban scenarios the randomness of user nodes' locations suggests the direct link may exist between all user nodes causing considerable degradation in the calculated achievable sum-rate.

In this paper and differently from the others, the performance of a multi-pair two-way FD AF massive MIMO relaying system with non-negligible direct links between all user pairs is evaluated. Considering non-negligible direct links between all user pairs constitutes a more general case for urban environments. The performance of the system is evaluated in terms of the achievable sum-spectral efficiency under two communication schemes. In the first scheme, user nodes attempt to capture the joint benifits of the relayed and direct links. In the second scheme, the users are assumed to be unaware of the presence of the direct link and so consider it as interference. All user nodes are equipped with multiple antennas, and the effect of spatial correlation between the antennas at each user node as well as the relay is investigated.

**Notations:** The lowercase and uppercase boldface letters denote column vectors and matrices, respectively. $\mathcal{CN}(0, \sigma^2)$ denotes the distribution of a circularly symmetric complex Gaussian variable with zero mean and variance $\sigma^2$. The symbols $(\cdot)^T$ and $(\cdot)^H$ are used to indicate transpose and conjugate transpose, respectively. Expectation, trace and determinant of a matrix are denoted by $\mathbb{E}(\cdot)$, tr$(\cdot)$ and det$(\cdot)$, respectively. $\mathbf{I}_M$ represents the $M \times M$ identity matrix and $\mathbf{0}_{M\times N}$ stands for an $M \times N$ matrix with all zero elements.

## II. System Model

### A. Communication Model

A multi-pair two-way massive MIMO relay system with one FD relay and K pairs of MIMO users is considered. All users operate in FD mode where each pair of users $(\mathrm{S}_{2k-1}, \mathrm{S}_{2k})$ attempt to exchange information with the aid of a massive

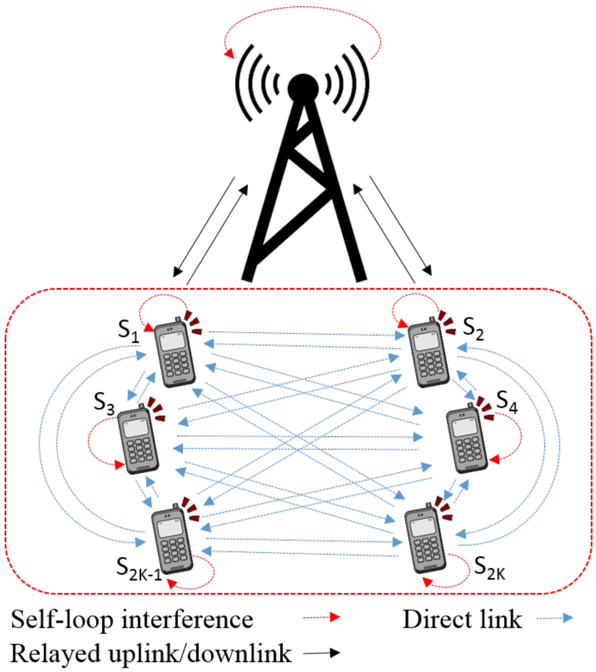


Fig. 1: A two-way full-duplex massive MIMO relay system

MIMO AF relay as shown in Fig. 1. Each user node is equipped with $N_s$ transmit antennas and $N_s$ receive antennas, while the relay is equipped with $N$ transmit antennas and $N$ receive antennas. In this work, the direct link between the users is assumed to be non-negligible which constitutes a general case where the communication between each user pair may be affected by other users. Moreover, we assume that the number of antennas at each user node is small, due to size limitation, relative to the relay which is equipped with a massive number of antennas, i.e., $N \gg 2KN_s$.

At time instant $n$, each user node $\mathrm{S}_k$ transmits an independent signal $\mathbf{x}_k(n) \in \mathbb{C}^{N_s}$, $k = 1, 2, \ldots, 2K$ and $\mathbb{E}(\mathbf{x}_k(n)\mathbf{x}_k(n)^H) = \mathbf{I}_{N_s}$ with a power of $P_s$. For simplicity, all user nodes are assumed to have the same transmit power constraint $P_s$. At the same time instant $n$, the relay broadcasts the signal $\mathbf{x}_R \in \mathbb{C}^N$ to all users with a power of $P_R$, i.e., $\mathrm{tr}(\mathbb{E}(\mathbf{x}_R(n)\mathbf{x}_R(n)^H)) = P_R$. At the relay, the transmitted signals from all users as well as the leaked signal from the relay transmit antennas are received. The leaked signal is the self-loop interference due to FD operation. Since the travelling distance between the relay transmit and receive antennas is short, the self-loop interference power is, in general, high with respect to other signals. As a result, despite the different self-interference cancellation techniques (e.g. passive, active analog and active digital methods), residual self-loop interference remains [10], [11]. The received signal at the relay, after self-interference cancellation, is given by

$$\mathbf{y}_R(n) = \sum_{k=1}^{2K} \sqrt{\frac{P_s}{N_s}} \mathbf{H}_{S_k R}\mathbf{x}_k(n) + \mathbf{H}_{RR}\mathbf{x}_R(n) + \mathbf{n}_R(n), \quad (1)$$

where $\mathbf{H}_{S_k R} \in \mathbb{C}^{N \times N_s}$ represents the channel response matrix from user node $\mathrm{S}_k$, $k = 1, 2, \ldots, 2K$, to the relay node, $\mathbf{H}_{RR} \in \mathbb{C}^{N \times N}$ denotes the self-loop channel response at the relay after self-interference cancellation and $\mathbf{H}_{RR}\mathbf{x}_R(n)$ corresponds to the residual self-loop interference at the relay, and $\mathbf{n}_R(n) \sim \mathcal{CN}(\mathbf{0}_{N\times 1}, \sigma_R^2 \mathbf{I}_N)$ denotes the complex Gaussian noise vector at the relay.

The AF protocol is used in order to reduce the computational complexity at the massive MIMO relay [9]. The broadcasted signal from the relay $\mathbf{x}_R(n)$ can be written as

$$\mathbf{x}_R(n) = \alpha \mathbf{W} \mathbf{y}_R(n - \delta), \quad (2)$$

where $\delta \geq 1$ represents the processing delay, $\alpha$ denotes the amplification factor set to satisfy the relay power constraint and $\mathbf{W}$ corresponds to the relay processing matrix which will be discussed later on. Similarly, the received signal, after self-interference cancellation, at user node $\mathrm{S}_k$ can be written as

$$\mathbf{y}_{S_k}(n) = \mathbf{H}_{RS_k}\mathbf{x}_R(n) + \sqrt{\frac{P_s}{N_s}}\boldsymbol{\Omega}_{k,k}\mathbf{x}_k(n) + \sum_{\substack{i=1\\ i\neq k}}^{2K} \sqrt{\frac{P_s}{N_s}}\boldsymbol{\Omega}_{i,k}\mathbf{x}_i(n) + \mathbf{n}_k(n), \quad (3)$$

where $\mathbf{H}_{RS_k}$ corresponds to the channel response matrix from the relay to user node $\mathrm{S}_k$, $\boldsymbol{\Omega}_{k,k}$ denotes the self-loop channel response at user node $\mathrm{S}_k$, $\boldsymbol{\Omega}_{i,k}$ represents the direct link from user node $\mathrm{S}_i$ to user node $\mathrm{S}_k$ and $\mathbf{n}_k(n) \sim \mathcal{CN}(\mathbf{0}_{N_s\times 1}, \sigma_k^2 \mathbf{I}_{N_s})$ denotes the complex Gaussian noise vector at user node $\mathrm{S}_k$.

Plugging (1) and (2) into (3), the received signal at user node $\mathrm{S}_k$ can be rewritten as

$$\mathbf{y}_{S_k}(n) = \alpha \mathbf{H}_{RS_k}\mathbf{W}\Bigg[\sqrt{\frac{P_s}{N_s}}\left(\mathbf{H}_{S_k R}\mathbf{x}_k(n-\delta) + \mathbf{H}_{S_{\bar{k}} R}\mathbf{x}_{\bar{k}}(n-\delta)\right) + \sum_{\substack{i=1\\ i\neq k,\bar{k}}}^{2K} \sqrt{\frac{P_s}{N_s}}\mathbf{H}_{S_i R}\mathbf{x}_i(n-\delta) + \mathbf{H}_{RR}\mathbf{x}_R(n-\delta) + \mathbf{n}_R(n-\delta)\Bigg] + \sqrt{\frac{P_s}{N_s}}\boldsymbol{\Omega}_{k,k}\mathbf{x}_k(n) + \sum_{\substack{i=1\\ i\neq k}}^{2K} \sqrt{\frac{P_s}{N_s}}\boldsymbol{\Omega}_{i,k}\mathbf{x}_i(n) + \mathbf{n}_k(n), \quad (4)$$

where $(\mathrm{S}_k, \mathrm{S}_{\bar{k}})$ correspond to a user pair attempting to exchange information with each other, such that $\bar{k} = k - 1$ if $k$ is even, and $\bar{k} = k + 1$ if $k$ is odd. The term $\alpha \mathbf{H}_{RS_k}\mathbf{W}\sqrt{\frac{P_s}{N_s}}\mathbf{H}_{S_k R}\mathbf{x}_k(n-\delta)$ represents the self-interference due to two-way relaying operation. By knowledge of the cross channel state information (CSI) between user nodes and the relay, i.e. $\mathbf{H}_{S_k R}$ and $\mathbf{H}_{RS_k}$, it can be perfectly cancelled [7]. In this paper, we assume that user nodes as well as the relay have perfect cross CSI. The CSI can be determined using training sequences and the length of the training sequence increases linearly with the number of antennas at the user nodes [9].

### *B. Channel Model*

Spatial correlation between the antennas both at the relay as well as the user nodes exists, in general, due to insufficient antenna spacing and the characteristics of the wireless channel. Following the Kronecker model, [9], [12], [13], the channel response matrices can be expressed as

$$\mathbf{H}_{AB} = \mathbf{R}_B^{\frac{1}{2}} \mathbf{W}_{AB} \mathbf{T}_A^{\frac{1}{2}}, \tag{5}$$

where $\mathbf{W}_{AB}$ consists of independent identically distributed (i.i.d.) Gaussian random entries with zero mean. $\mathbf{R}_B$ and $\mathbf{T}_A$ are nonnegative definite matrices, representing the receive antenna correlation at node B and the transmit antenna correlation at node A, respectively. Without loss of generality, $\mathbf{R}_B$ and $\mathbf{T}_A$ are assumed to be normalized such that tr$(\mathbf{R}_{S_i}) =$ tr$(\mathbf{T}_{S_i}) = N_s$, and tr$(\mathbf{R}_R) =$ tr$(\mathbf{T}_R) = N$. Moreover, the spectral radius of $\mathbf{R}_{S_i}$ and $\mathbf{T}_{S_i}$ is bounded by a constant [13], [14]. The Kronecker model presented above is applicable to MIMO channels when there is a physical seperation between the transmitting and receiving antennas [13], which is true for most wireless communication systems. We also assume that different sets of antennas are used for transmission and reception at each node, and so the Kronecker model is valid to describe the self-loop channel as well.

### C. Relay Processing

At the relay, a low complexity maximum ratio combining/maximum ratio transmission (MRC/MRT) scheme is adopted. Using (1) and (2), and the aforementioned assumption for the cross CSI, the relay power constraint is represented as

$$\alpha^2 \left[ \frac{P_s}{N_s} \sum_{k=1}^{2K} \text{tr}\left(\mathbf{W}\mathbf{H}_{S_k R}\mathbf{H}_{S_k R}^H \mathbf{W}^H\right) + \frac{P_R}{N} \text{tr}\left(\mathbf{W}\mathbb{E}\left(\mathbf{H}_{RR}\mathbf{H}_{RR}^H\right)\mathbf{W}^H\right) + \sigma_R^2 \text{tr}\left(\mathbf{W}\mathbf{W}^H\right)\right] = P_R, \tag{6}$$

Based on the Kronecker model presented in II-B, and as shown in [15]

$$\mathbb{E}\left(\mathbf{H}_{RR}\mathbf{H}_{RR}^H\right) = \sigma_{\text{LIr}}^2 N \mathbf{R}_R, \tag{7}$$

where $\sigma_{\text{LIr}}^2$ denotes the variance of the residual self-loop interference. From (6) and (7), the amplification factor $\alpha$ can be written as (8) at the top of next page. Moreover, the MRC/MRT processing matrix $\mathbf{W}$ can be represented as [16]

$$\mathbf{W} = \mathbf{H}_{RD}^H \mathbf{T} \mathbf{H}_{SR}^H, \tag{9}$$

where $\mathbf{H}_{RD} = \left[\mathbf{H}_{RS_1}^T \mathbf{H}_{RS_2}^T \cdots \mathbf{H}_{RS_{2K}}^T\right]^T \in \mathbb{C}^{2KN_s \times N}$ corresponds to the channel response matrix from the relay to all user nodes, $\mathbf{H}_{SR} = \left[\mathbf{H}_{S_1 R}\mathbf{H}_{S_2 R} \cdots \mathbf{H}_{S_{2K} R}\right] \in \mathbb{C}^{N \times 2KN_s}$ corresponds to the channel response matrix from all user nodes to the relay node and $\mathbf{T}$ is a block permutation matrix such that $\mathbf{T} = \text{diag}(\mathbf{T}_1, \mathbf{T}_2, \cdots, \mathbf{T}_K)$, and $\mathbf{T}_k$ has the form

$$\mathbf{T}_k = \begin{bmatrix} \mathbf{0}_{N_s \times N_s} & \widetilde{\mathbf{T}}_{2k-1} \\ \widetilde{\mathbf{T}}_{2k} & \mathbf{0}_{N_s \times N_s} \end{bmatrix}, \tag{10}$$

where $\widetilde{\mathbf{T}}_{2k-1}, \widetilde{\mathbf{T}}_{2k} \in \mathbb{C}^{N_s \times N_s}$ are permutation matrices representing information exchange with binary elements between the $k^{th}$ user pair $(2k-1, 2k)$.

## III. Achievable Sum-Rate

The achievable sum-rate of the FD two-way relaying system described above under perfect cross CSI can be written as

$$R_{sum} = \sum_{k=1}^{2K} R_k, \tag{11}$$

where $R_k$ corresponds to the achievable rate at user node $\text{S}_k$. Using (4), the received signal at user node $\text{S}_k$ is represented as (12) at the top of next page. We then evaluate the performance of the system with two reception schemes at the user nodes: joint relayed and direct link (RDL) reception, and the direct link interference (DLI).

In the RDL mode, each user node attempts to capture both the relayed and direct links. As shown in (12), the relayed and direct links arrive at user node $\text{S}_k$ at different time instances due to the processing delay $\delta$ at the relay. In order to eliminate the inter-symbol interference (ISI) caused by this delay, frequency domain equalization is performed at the user nodes. For this purpose, a cyclic prefix (CP) transmission, of length $\tau_{CP}(\geq \delta)$, is required at the trasmitting side for each $T$ symbols. In this work, we assume all channel gains remain constant over a block duration of $T + \tau_{CP}$ symbols. Using (7), (12), and similar mathematical manipulations as in [4], [17], the achievable rate at user node $\text{S}_k$ can be approximated by (13) at the top of next page. In (13), the terms $\sigma_{\text{LIr}}^2 P_s \mathbf{R}_{S_k}$, $(2K-2)\,\sigma_{IUI}^2 P_s \mathbf{R}_{S_k}$ and $\sigma_D^2 P_s \mathbf{R}_{S_k}$ correspond to the residual self-loop interference, the interfering direct links and the direct link from the paired user at $\text{S}_k$, respectively, and can be derived using the result in (7). In the DLI mode, the user nodes treat the direct link as interference and the achievable rate $R_k$ is given as (14) at the top of next page.

## IV. Deterministic Sum-Rate

The analytical derivations of the achievable sum-rate with closed-form deterministic expressions are presented in this section. First, a useful lemma is presented.

**Lemma 1** Let $\mathbf{P} \in \mathbb{C}^{\mathbb{N} \times \mathbb{N}}$ be a deterministic matrix with bounded spectral norm, i.e. $\|\mathbf{P}\| < \infty$; $\mathbf{A}$, $\mathbf{B} \in \mathbb{C}^{M \times N}$ be mutually independent random matrices with independent identically distributed (i.i.d.) Gaussian values of zero mean and variance $\beta^2$ such that $\mathbf{A}, \mathbf{B} \sim \mathcal{CN}\left(\mathbf{0}, \beta^2 \mathbf{I}_M \otimes \mathbf{I}_N\right)$. Then the following is true [18]

$$\tfrac{1}{N}\mathbf{A}\mathbf{P}\mathbf{A}^H \xrightarrow[N\to\infty]{a.s.} \tfrac{1}{N}\beta^2 \text{tr}(\mathbf{P})\, \mathbf{I}_M$$

$$\tfrac{1}{N}\mathbf{A}\mathbf{P}\mathbf{B}^H \xrightarrow[N\to\infty]{a.s.} \mathbf{0}_M$$

Defining $\mathbf{Z}_{k1}$, $\mathbf{Z}_{k2}$, $\mathbf{Z}_{k3}$ and $\mathbf{Z}_{k4}$ by

$$\mathbf{Z}_{k1} = \left(\mathbf{H}_{RS_k}\mathbf{W}\mathbf{H}_{S_{\bar{k}}R}\right)\left(\mathbf{H}_{RS_k}\mathbf{W}\mathbf{H}_{S_{\bar{k}}R}\right)^H,$$
$$\mathbf{Z}_{k2} = \sum_{\substack{i=1 \\ i\neq k,\bar{k}}}^{2K} \left(\mathbf{H}_{RS_k}\mathbf{W}\mathbf{H}_{S_i R}\right)\left(\mathbf{H}_{RS_k}\mathbf{W}\mathbf{H}_{S_i R}\right)^H,$$
$$\mathbf{Z}_{k3} = \left(\mathbf{H}_{RS_k}\mathbf{W}\right)\mathbf{R}_R\left(\mathbf{H}_{RS_k}\mathbf{W}\right)^H,$$
$$\mathbf{Z}_{k4} = \left(\mathbf{H}_{RS_k}\mathbf{W}\right)\left(\mathbf{H}_{RS_k}\mathbf{W}\right)^H.$$

Then, using $N^3$ and $\frac{1}{N^4}$ as common factors, (14) can be rewritten as (15) on the next page.

$$\alpha = \sqrt{\frac{P_R}{\frac{P_s}{N_s}\underbrace{\sum_{k=1}^{2K}\operatorname{tr}\left(\mathbf{W}\mathbf{H}_{S_kR}\mathbf{H}_{S_kR}^H\mathbf{W}^H\right)}_{D1} + \sigma_{\mathrm{LIr}}^2 P_R \underbrace{\operatorname{tr}\left(\mathbf{W}\mathbf{R}_R\mathbf{W}^H\right)}_{D2} + \sigma_R^2 \underbrace{\operatorname{tr}\left(\mathbf{W}\mathbf{W}^H\right)}_{D3}}}, \tag{8}$$

$$\mathbf{y}_{S_k}(n) = \underbrace{\alpha\sqrt{\frac{P_s}{N_s}}\mathbf{H}_{RS_k}\mathbf{W}\mathbf{H}_{S_{\bar{k}}R}\mathbf{x}_{\bar{k}}(n-\delta)}_{\text{Desired relayed signal}} + \underbrace{\alpha\sqrt{\frac{P_s}{N_s}}\mathbf{H}_{RS_k}\mathbf{W}\sum_{\substack{i=1\\ i\neq k,\bar{k}}}^{2K}\mathbf{H}_{S_iR}\mathbf{x}_i(n-\delta) + \sum_{\substack{i=1\\ i\neq k,\bar{k}}}^{2K}\sqrt{\frac{P_s}{N_s}}\mathbf{\Omega}_{i,k}\mathbf{x}_i(n)}_{\text{Inter-user interference}}$$
$$+ \underbrace{\sqrt{\frac{P_s}{N_s}}\mathbf{\Omega}_{\bar{k},k}\mathbf{x}_{\bar{k}}(n)}_{\text{Direct link}} + \underbrace{\sqrt{\frac{P_s}{N_s}}\mathbf{\Omega}_{k,k}\mathbf{x}_k(n) + \alpha\mathbf{H}_{RS_k}\mathbf{W}\mathbf{H}_{RR}\mathbf{x}_R(n-\delta)}_{\text{Residual self-loop interference}} + \underbrace{\alpha\mathbf{H}_{RS_k}\mathbf{W}\mathbf{n}_R(n-\delta) + \mathbf{n}_k(n)}_{\text{Noise}}, \tag{12}$$

$$R_k^{RDL} \approx \frac{T}{T+\tau_{CP}}\log_2\det\Bigg(\mathbf{I}_{N_s} + \left[\frac{\alpha^2 P_s}{N_s}\left(\mathbf{H}_{RS_k}\mathbf{W}\mathbf{H}_{S_{\bar{k}}R}\right)\left(\mathbf{H}_{RS_k}\mathbf{W}\mathbf{H}_{S_{\bar{k}}R}\right)^H + \sigma_D^2 P_s\mathbf{R}_{S_k}\right]$$
$$\times\left[\frac{\alpha^2 P_s}{N_s}\sum_{\substack{i=1\\ i\neq k,\bar{k}}}^{2K}\left(\mathbf{H}_{RS_k}\mathbf{W}\mathbf{H}_{S_iR}\right)\left(\mathbf{H}_{RS_k}\mathbf{W}\mathbf{H}_{S_iR}\right)^H + \alpha^2\sigma_{\mathrm{LIr}}^2 P_R\left(\mathbf{H}_{RS_k}\mathbf{W}\right)\mathbf{R}_R\left(\mathbf{H}_{RS_k}\mathbf{W}\right)^H\right.$$
$$\left.+\alpha^2\sigma_R^2\left(\mathbf{H}_{RS_k}\mathbf{W}\right)\left(\mathbf{H}_{RS_k}\mathbf{W}\right)^H + \sigma_{\mathrm{LIr}}^2 P_s\mathbf{R}_{S_k} + (2K-2)\,\sigma_{\mathrm{IUI}}^2 P_s\mathbf{R}_{S_k} + \sigma_k^2\mathbf{I}_{N_s}\right]^{-1}\Bigg) \tag{13}$$

$$R_k^{DLI} = \log_2\det\Bigg(\mathbf{I}_{N_s} + \frac{\alpha^2 P_s}{N_s}\left(\mathbf{H}_{RS_k}\mathbf{W}\mathbf{H}_{S_{\bar{k}}R}\right)\left(\mathbf{H}_{RS_k}\mathbf{W}\mathbf{H}_{S_{\bar{k}}R}\right)^H\left[\frac{\alpha^2 P_s}{N_s}\sum_{\substack{i=1\\ i\neq k,\bar{k}}}^{2K}\left(\mathbf{H}_{RS_k}\mathbf{W}\mathbf{H}_{S_iR}\right)\left(\mathbf{H}_{RS_k}\mathbf{W}\mathbf{H}_{S_iR}\right)^H\right.$$
$$\left.+\alpha^2\sigma_{\mathrm{LIr}}^2 P_R\left(\mathbf{H}_{RS_k}\mathbf{W}\right)\mathbf{R}_R\left(\mathbf{H}_{RS_k}\mathbf{W}\right)^H + \alpha^2\sigma_R^2\left(\mathbf{H}_{RS_k}\mathbf{W}\right)\left(\mathbf{H}_{RS_k}\mathbf{W}\right)^H + \sigma_{\mathrm{LIr}}^2 P_s\mathbf{R}_{S_k} + \left((2K-2)\,\sigma_{\mathrm{IUI}}^2 + \sigma_D^2\right)P_s\mathbf{R}_{S_k} + \sigma_k^2\mathbf{I}_{N_s}\right]^{-1}\Bigg) \tag{14}$$

$$R_k^{DLI} = \log_2\det\Bigg(\mathbf{I}_{N_s} + \frac{P_s}{N_s}(N^{\frac{3}{2}}\alpha)^2\left(\frac{1}{N^4}\mathbf{Z}_{k1}\right)\times\left[\frac{P_s}{N_s}(N^{\frac{3}{2}}\alpha)^2\left(\frac{1}{N^4}\mathbf{Z}_{k2}\right) + (N^{\frac{3}{2}}\alpha)^2\sigma_{\mathrm{LIr}}^2 P_R\left(\frac{1}{N^4}\mathbf{Z}_{k3}\right)\right.$$
$$\left.+(N^{\frac{3}{2}}\alpha)^2\sigma_R^2\left(\frac{1}{N^4}\mathbf{Z}_{k4}\right) + \frac{\sigma_{\mathrm{LIr}}^2 P_s}{N}\mathbf{R}_{S_k} + \frac{\left((2K-2)\,\sigma_{\mathrm{IUI}}^2 + \sigma_D^2\right)P_s}{N}\mathbf{R}_{S_k} + \frac{\sigma_k^2}{N}\mathbf{I}_{N_s}\right]^{-1}\Bigg) \tag{15}$$

Using (9), the MRC/MRT processing matrix $\mathbf{W}$ can be written as

$$\begin{aligned}\mathbf{W} &= \sum_{k=1}^{K}\left(\mathbf{H}_{RS_{2k-1}}^H\mathbf{H}_{RS_{2k}}^H\right)\mathbf{T}_k\left(\mathbf{H}_{S_{2k-1}R}\mathbf{H}_{S_{2k}R}\right)^H\\ &= \sum_{k=1}^{K}\left(\mathbf{H}_{RS_{2k}}^H\widetilde{\mathbf{T}}_{2k}\mathbf{H}_{S_{2k-1}R}^H + \mathbf{H}_{RS_{2k-1}}^H\widetilde{\mathbf{T}}_{2k-1}\mathbf{H}_{S_{2k}R}^H\right)\\ &= \sum_{k=1}^{2K}\mathbf{H}_{RS_{\bar{k}}}^H\widetilde{\mathbf{T}}_{\bar{k}}\mathbf{H}_{S_kR}^H,\end{aligned} \tag{16}$$

From (15), and to continue with the analysis, it is clear that the terms $N^{\frac{3}{2}}\alpha$, $\mathbf{Z}_{k1}$, $\mathbf{Z}_{k2}$, $\mathbf{Z}_{k3}$ and $\mathbf{Z}_{k4}$ need to be deterministically approximated. The terms including $D_1$, $D_2$ and $D_3$ will be investigated seperately in the following. Using (16), $\operatorname{tr}(\mathbf{AB}) = \operatorname{tr}(\mathbf{BA})$, we have

$$\begin{aligned}\frac{1}{N^3}D_1 &= \frac{1}{N^3}\sum_{k=1}^{2K}\operatorname{tr}\left(\mathbf{W}\mathbf{H}_{S_kR}\mathbf{H}_{S_kR}^H\mathbf{W}^H\right)\\ &= \sum_{i=1}^{2K}\sum_{j=1}^{2K}\sum_{k=1}^{2K}\operatorname{tr}\left(\frac{\mathbf{H}_{RS_{\bar{k}}}\mathbf{H}_{RS_{\bar{j}}}^H}{N}\widetilde{\mathbf{T}}_{\bar{j}}\frac{\mathbf{H}_{S_jR}^H\mathbf{H}_{S_iR}}{N}\frac{\mathbf{H}_{S_iR}^H\mathbf{H}_{S_kR}}{N}\widetilde{\mathbf{T}}_{\bar{k}}^H\right)\end{aligned} \tag{17}$$

Using the Kronecker model defined in (5) and the result shown in lemma 1 along with the fact that all transmit and receive correlation matrices are Hermitian matrices, the following can be obtained

$$\frac{\mathbf{H}_{RS_k}\mathbf{H}_{RS_j}^H}{N} \xrightarrow[N\to\infty]{a.s.} \frac{\mathrm{tr}(\mathbf{T}_R)}{N}\beta^2\delta_{kj}\mathbf{R}_{S_k} \tag{18}$$

$$\frac{\mathbf{H}_{S_kR}^H\mathbf{H}_{S_jR}}{N} \xrightarrow[N\to\infty]{a.s.} \frac{\mathrm{tr}(\mathbf{R}_R)}{N}\beta^2\delta_{kj}\mathbf{T}_{S_k} \tag{19}$$

where, $\delta_{kj} = \begin{cases} 1 & k = j \\ 0 & k \neq j \end{cases}$.

Recalling from Section II-B that $\mathrm{tr}(\mathbf{T}_R) = \mathrm{tr}(\mathbf{R}_R) = N$, and substituting (18) and (19) into (17), we have

$$\frac{1}{N^3}D_1 \xrightarrow[N\to\infty]{a.s.} \beta^6\sum_{k=1}^{2K}\mathrm{tr}\left(\mathbf{R}_{S_k}\widetilde{\mathbf{T}}_k\mathbf{T}_{S_k}^2\widetilde{\mathbf{T}}_k^H\right) \tag{20}$$

Note that the value of the approximations used in (17) will be nonzero only when $i = j = k$ assuming independency between a user's channel and all other users, resulting in the single summation in (20). For $D_2$, using (16) and $\mathrm{tr}(\mathbf{AB}) = \mathrm{tr}(\mathbf{BA})$, we have

$$\begin{aligned}\frac{1}{N^3}D_2 &= \frac{1}{N^3}\mathrm{tr}\left(\mathbf{W}\mathbf{R}_R\mathbf{W}^H\right)\\ &= \frac{1}{N}\sum_{j=1}^{2K}\sum_{k=1}^{2K}\mathrm{tr}\left(\frac{\mathbf{H}_{RS_{\bar{k}}}\mathbf{H}_{RS_{\bar{j}}}^H}{N}\widetilde{\mathbf{T}}_{\bar{j}}\frac{\mathbf{H}_{S_jR}^H\mathbf{R}_R\mathbf{H}_{S_kR}}{N}\widetilde{\mathbf{T}}_{\bar{k}}^H\right)\end{aligned} \tag{21}$$

Substituting (18) and (19) into (21), and with a similar procedure to (20), the following is obtained

$$\frac{1}{N^3}D_2 \xrightarrow[N\to\infty]{a.s.} \frac{\beta^4}{N}\frac{\mathrm{tr}\left(\mathbf{R}_R^2\right)}{N}\sum_{k=1}^{2K}\mathrm{tr}\left(\mathbf{R}_{S_k}\widetilde{\mathbf{T}}_k\mathbf{T}_{S_k}\widetilde{\mathbf{T}}_k^H\right) \tag{22}$$

As for $D_3$, using similar mathematical manipulations, we get

$$\frac{1}{N^3}D_3 \xrightarrow[N\to\infty]{a.s.} \frac{\beta^4}{N}\sum_{k=1}^{2K}\mathrm{tr}\left(\mathbf{R}_{S_k}\widetilde{\mathbf{T}}_k\mathbf{T}_{S_k}\widetilde{\mathbf{T}}_k^H\right) \tag{23}$$

By substituting (20), (22) and (23) into (8), the deterministic approximation of $N^{\frac{3}{2}}\alpha$ is given by $\bar{\alpha}$ as

$$\bar{\alpha} = \sqrt{\frac{P_R}{\frac{P_s\beta^6}{N_s}F_1 + \left(\frac{\sigma_{\mathrm{LIr}}^2 P_R\beta^4}{N}\frac{\mathrm{tr}\left(\mathbf{R}_R^2\right)}{N} + \frac{\sigma_R^2\beta^4}{N}\right)F_2}} \tag{24}$$

where, $F_1 = \sum_{k=1}^{2K}\mathrm{tr}\left(\mathbf{R}_{S_k}\widetilde{\mathbf{T}}_k\mathbf{T}_{S_k}^2\widetilde{\mathbf{T}}_k^H\right)$,

and, $F_2 = \sum_{k=1}^{2K}\mathrm{tr}\left(\mathbf{R}_{S_k}\widetilde{\mathbf{T}}_k\mathbf{T}_{S_k}\widetilde{\mathbf{T}}_k^H\right)$.

Following the same procedure used in getting $\bar{\alpha}$, and to avoid redundancy, the following can be directly obtained

$$\frac{1}{N^4}\mathbf{Z}_{k1} \xrightarrow[N\to\infty]{a.s.} \beta^8\left(\mathbf{R}_{S_k}\widetilde{\mathbf{T}}_k\mathbf{T}_{S_k}^2\widetilde{\mathbf{T}}_k^H\mathbf{R}_{S_k}\right) \tag{25}$$

$$\frac{1}{N^4}\mathbf{Z}_{k2} \xrightarrow[N\to\infty]{a.s.} \mathbf{0}_{N_s} \tag{26}$$

$$\frac{1}{N^4}\mathbf{Z}_{k3} \xrightarrow[N\to\infty]{a.s.} \frac{\beta^6}{N}\frac{\mathrm{tr}\left(\mathbf{R}_R^2\right)}{N}\left(\mathbf{R}_{S_k}\widetilde{\mathbf{T}}_k\mathbf{T}_{S_k}\widetilde{\mathbf{T}}_k^H\mathbf{R}_{S_k}\right) \tag{27}$$

$$\frac{1}{N^4}\mathbf{Z}_{k4} \xrightarrow[N\to\infty]{a.s.} \frac{\beta^6}{N}\left(\mathbf{R}_{S_k}\widetilde{\mathbf{T}}_k\mathbf{T}_{S_k}\widetilde{\mathbf{T}}_k^H\mathbf{R}_{S_k}\right) \tag{28}$$

Plugging (24)–(28) into (15), the deterministic rate $\overline{R}_k^{DLI}$ is given by

$$\begin{aligned}\bar{R}_k^{DLI} &= \log_2\det\Bigg(\mathbf{I}_{N_s} + \frac{P_s\bar{\alpha}^2\beta^8}{N_s}\left(\mathbf{R}_{S_k}\widetilde{\mathbf{T}}_k\mathbf{T}_{S_k}^2\widetilde{\mathbf{T}}_k^H\mathbf{R}_{S_k}\right)\\ &\times\Bigg[\left(\frac{\bar{\alpha}^2\sigma_{\mathrm{LIr}}^2P_R\beta^6}{N}\frac{\mathrm{tr}\left(\mathbf{R}_R^2\right)}{N} + \frac{\bar{\alpha}^2\sigma_R^2\beta^6}{N}\right)\left(\mathbf{R}_{S_k}\widetilde{\mathbf{T}}_k\mathbf{T}_{S_k}\widetilde{\mathbf{T}}_k^H\mathbf{R}_{S_k}\right)\\ &+ \frac{\sigma_{\mathrm{LIr}}^2P_s}{N}\mathbf{R}_{S_k} + \frac{\left((2K-2)\,\sigma_{\mathrm{IUI}}^2 + \sigma_D^2\right)P_s}{N}\mathbf{R}_{S_k} + \frac{\sigma_k^2}{N}\mathbf{I}_{N_s}\Bigg]^{-1}\Bigg)\end{aligned} \tag{29}$$

It is noted in (29) that as the number of relay antennas grows large, the inter-user interference (IUI) which is the term including $\mathbf{Z}_{k2}$ is completely eliminated. Moreover, the derived expression for the rate is applicable to arbitrary transmit and receive correlation matrices at the relay and user nodes.

## V. Results and Discussions

In this section, we analyze the effect of different system parameters on the sum-rate using Monte-Carlo simulations. The exponential model is adopted in this work to represent the receive antenna correlation at the relay and user nodes, which is applicable to uniform linear arrays [19], such that

$$\mathbf{R}_R = \begin{bmatrix} 1 & \rho_o & \rho_o^2 & \rho_o^3 & \cdots & \rho_o^{N-1} \\ \rho_o & 1 & \rho_o & \rho_o^2 & \cdots & \rho_o^{N-2} \\ \rho_o^2 & \rho_o & 1 & \rho_o & \cdots & \rho_o^{N-3} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho_o^{N-1} & \rho_o^{N-2} & \rho_o^{N-3} & \rho_o^{N-4} & \cdots & 1 \end{bmatrix}, \tag{30}$$

where $\rho_o$ denotes the correlation coefficient at the relay. The following setting is used in all simulations, unless otherwise indicated: $P_s = 0dB, P_R = 10dB$, $\rho_o = 0.7$, $T = 500$, $\tau_{CP} = 20$, $\sigma_{LIr}^2 = 0.05$ and $\sigma_R^2 = \sigma_k^2 = \sigma_{\mathrm{IUI}}^2 = 1$. The users antenna correlation coefficient $\rho_s$ is set to 0.1, since the number of antennas at the user nodes is small compared to the massive number at the relay, and a much smaller spacing between the user antennas is required for a relatively low antenna correlation [20].

Figure 2 plots the achievable sum-rate versus the number of relay antennas for the deterministic and simulation results. It is shown that the analytical results agree with the simulation results, validating the theoretical developments. The gap between analytical and simulation results is seen to decrease with increasing $N$, which is logical, as the analysis is based on the existance of a largescale antenna array at the relay.

Figures 3 and 4 show the achievable sum-rate of the two-way FD relaying system with a uniform linear array configuration at the relay and user nodes versus $\rho_o$ and $\rho_s$, respectively. It is clear that the achievable rate decreases as $\rho_o$ increases, i.e. the array size decreases, which is a logical outcome as the effective dimensionality of the relay array decreases with increasing the spatial correlation among its elements. Moreover, the difference in the system's performance for different numbers of user pairs decreases with increasing $\rho_o$.

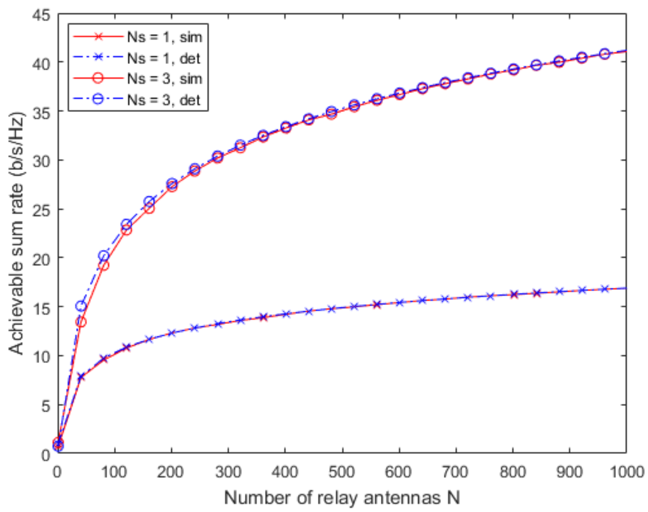


Fig. 2: Achievable sum-rate versus N for deterministic and simulation results, DLI, $\sigma_D^2 = 1$, $K = 1$

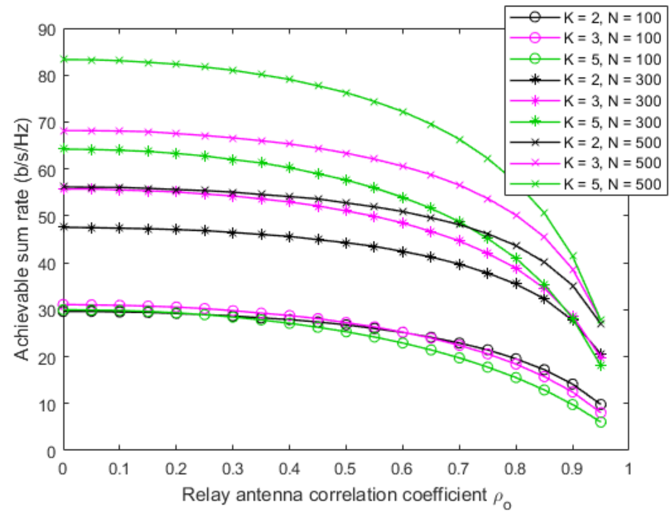


Fig. 3: Achievable sum-rate versus relay correlation for DLI, $N_s = 3$, $\sigma_D^2 = 1$

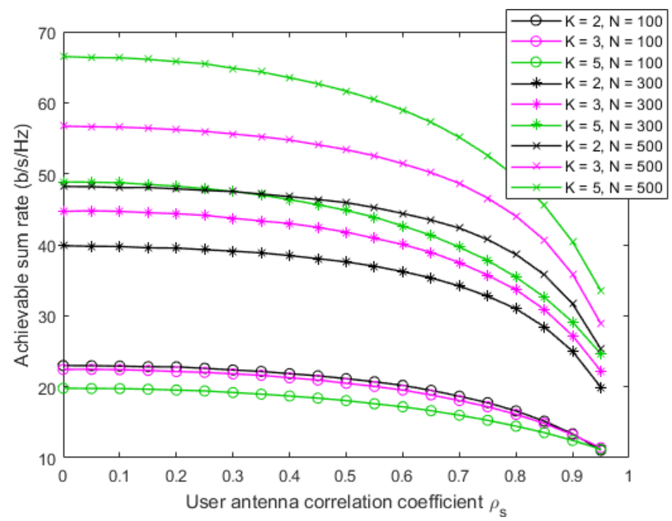


Fig. 4: Achievable sum-rate versus user correlation for DLI, $N_s = 3$, $\sigma_D^2 = 1$

This is due to the fact that the intermediate node, which is the relay, becomes the limiting factor of the achievable rate, and so the impact of the increase in the dimensionality of the receivers (user nodes) becomes less significant. It is noted that for $N = 100$, the achievable rate in case of $K = 5$ is actually less than that for $K = 2$ especially for high values of correlation between the relay antennas, which shows that for this value of $N$ the relay is uncapable of effectively mitigating the IUI using the linear MRC/MRT processing technique. In Fig. 4, a similar behaviour is seen as in the case of varying $\rho_o$. However, this time for $N = 100$, the achievable rate for $K = 5$ user pairs reaches the same value as that for $K = 2$ when $\rho_s$ increases. On one hand, the decrease in the effective dimension of the user receive array decreases its capacity for communication. On the other hand, the decrease in this dimension helps the relay to focus power in the directions of the receivers and so the decrease in capacity is less severe for the higher number of users case. For comparison between the two reception modes discussed, the achievable sum-rates of the two-way FD relay system for the DLI mode are shown in Figs. 5 and 6, and for the RDL mode shown in Figs. 7 and 8 with weak and strong direct link gains, respectively. It is clear that increasing the number of antennas at the user nodes and/or the number of user pairs increases the sum-rate, provided the number of relay antennas is sufficiently large for the mitigation of added interference. Figures 5 and 7 show that the two modes perform about the same if the direct link from the paired user to each user node is relatively weak. However, under the existance of a strong direct link, Figs. 6 and 8 show that the RDL mode outperforms the DLI mode for the selected communication block length. In addition, the decrease in the achievable rate with increasing the number of relay antennas, shown in Fig. 8, especially for $N_s = 3$ is because the effective relay array dimensionality for $N = 1 \rightarrow 150$ is insufficient to mitigate the IUI between the antennas at all user nodes, and so the dominant part of the relayed link is actually the IUI. That is, the relay here acts as a source of delivering the IUI to the user nodes. This indicates that for the simulated scenario parameters and $N < 150$, it is better to omit the relay and use device-to-device (D2D) communication.

Similarly, the achievable sum-rates of the system versus the number of user pairs $K$ for the DLI mode are shown in Figs. 9 and 10, and for the RDL mode shown in Figs. 11 and 12 with weak and strong direct link gains, respectively. Obviously, for a higher direct link gain, the achievable rate for the DLI mode decreases while that for the RDL mode increases for all values of K. It is noted that for $N_s = 3$, a slight increase in the performance is seen for the RDL mode over the DLI mode, with a weak direct link gain, for high values of $K$, and especially for a lower number of relay

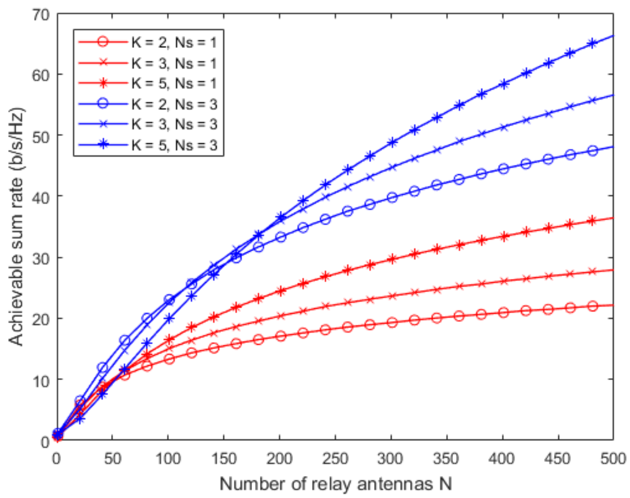


Fig. 5: Achievable sum-rate for DLI, $\sigma_D^2 = 1$

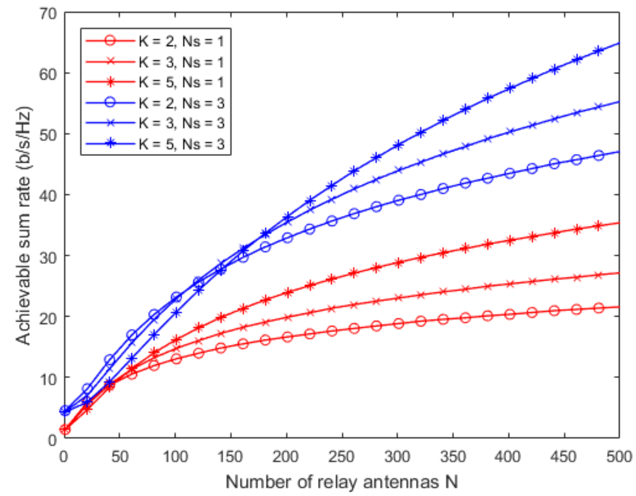


Fig. 7: Achievable sum-rate for RDL, $\sigma_D^2 = 1$

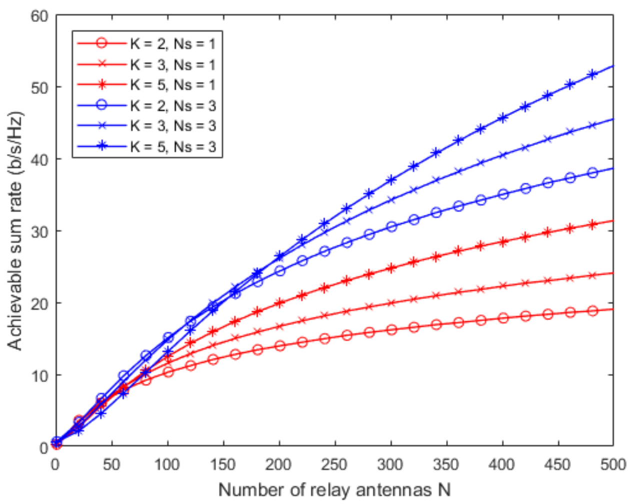


Fig. 6: Achievable sum-rate for DLI, $\sigma_D^2 = 20$

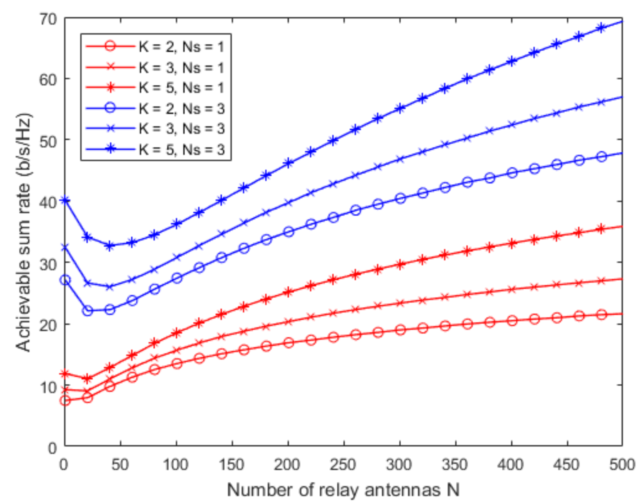


Fig. 8: Achievable sum-rate for RDL, $\sigma_D^2 = 20$

antennas. This is because of the less quality of the relayed link carrying additional IUI, and thus increasing the significance of capturing the direct link. Moreover, Figs. 11 and 12 show that the maximum rate of the RDL mode is achieved at a higher value of $K$ for the case of a strong direct link gain. This indicates that the system's capability to serve more users with a specific target achievable sum-rate increases.

## VI. Conclusions

This paper evaluates the performance of a multi-pair two-way FD massive MIMO relaying system with correlated antennas at the relay and user nodes. The performance of the system is evaluated in terms of the achievable sum-rate under two communication modes. It was found that the RDL mode outperforms the DLI mode in the presence of a strong direct link, while the performance of the two modes is about the same if the direct link between paired users is the same as for other interfering users. Moreover, increasing the number of user pairs increases the sum-rate, provided that the effective dimensionality of the relay array is sufficiently large.

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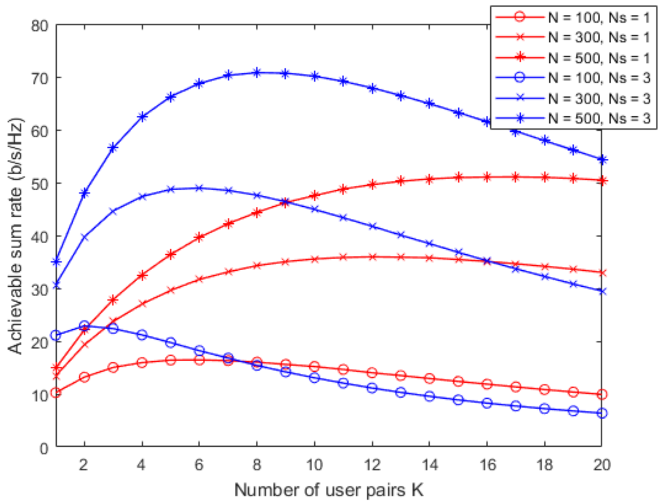


Fig. 9: Achievable sum-rate for DLI, $\sigma_D^2 = 1$

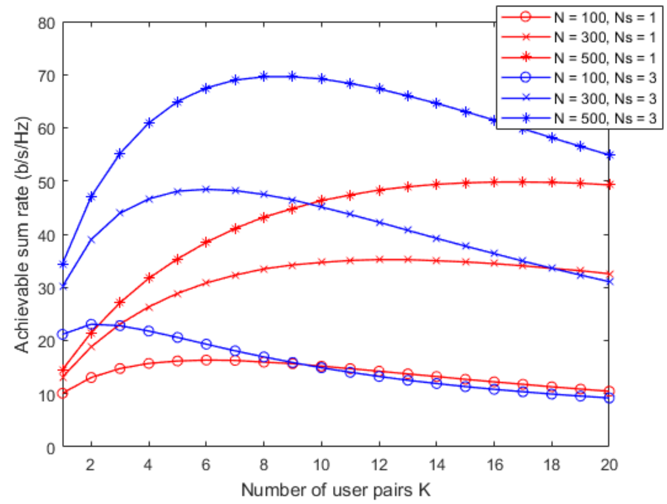


Fig. 11: Achievable sum-rate for RDL, $\sigma_D^2 = 1$

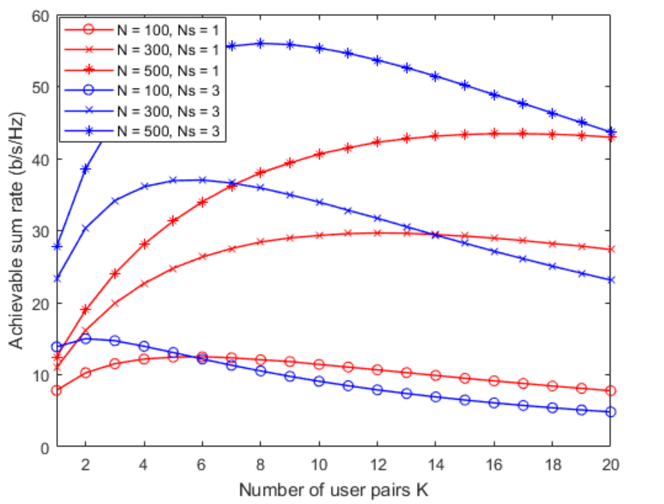


Fig. 10: Achievable sum-rate for DLI, $\sigma_D^2 = 20$

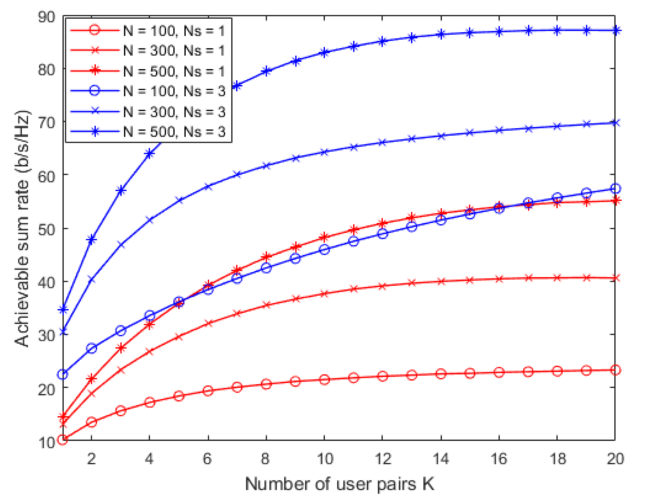


Fig. 12: Achievable sum-rate for RDL, $\sigma_D^2 = 20$


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