MABSDairy: A MULTISCALE AGENT BASED SIMULATION OF A DAIRY HERD

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ABSTRACT

Agent based simulation models (ABSM) provide individual information about an agent in a multiscale system problem. Here, we developed a multiscale agent based simulation model of a dairy herd (MABSDairy) by considering several layers of detailed information (i.e., individual animal, overall population and farmer's decision) to make optimal decisions. Previously, dynamic programming (DP) has been widely studied to find the optimal cow replacement policies, but DP models are computationally intensive and might not be practical for daily decision making for a farmer. Hence, we developed new ABSM for individual animals on a previously run DP model to provide fast and accurate predictions of nonlinear and inter correlated parameters. The results show that the model can be applied to estimate critical parameters for management decisions. Overall, the MABSDairy presents an adaptive simulation tool where different diseases and different management policies may be included in the future.

Keywords: multiscale agent based model, stochastic model, dynamic programing, dairy management, and milk yield.

1 INTRODUCTION

The dairy sector is a major agricultural part of the US economy and according to the USDA, farms with more than 500 milking cows now account for 63% of the milk supply in the United States (USDA-NASS, 2012), up from 39% a decade ago (USDA-NASS, 2002). Although a modern farm is equipped with modern production facilities, farmers still face substantial challenges in making decisions about events like culling, replacing cows, disease supervision, when to cull during infectious diseases or other production related diseases, etc. All these decisions are related to the market price of produced milk, carcass value, heifer replacement value, and feed cost (Delorenzo and Thomas, 1996).

From the modeling point of view, a dairy cow consists of several layers of information: individual animal groups, daily activities of each group, milk yield, replacement, reproduction, disease management, and herd progress measurements. Also, different time scales exist in different processes, making it a multiscale problem. For example, herd progress is measured daily, culling decisions are made weekly or monthly; lactation involves several processes having discrete time scales, and management decision time varies with strategies undertaken. Over the past decades, several studies have considered the dairy herd simulation problem as a modeling problem. Dynamic programming (DP) is an optimization technique, and has been widely studied over the past several decades (van Arendonk, 1985; Kristensen, 2003; Bar et al. 2008). However, DP models are complex in nature and computationally intensive and might not be practical for daily decision making.

SpringSim-ANSS 2017, April 23-26, Virginia Beach, VA, USA

Due to the limitations of implementing DP in real herds, researchers have developed individual based models of a dairy herd, e.g., JohneSim (Kudahl et al. 2007), and agent based models (e.g., Davidson et al. (2012), Robins et al. (2015), Al-Mamun et al. (2016), and Kirkeby et al. (2016)). However, recent models have been developed on specific problem hypotheses related to different diseases where only minimal information regarding a dairy herd is considered. Davidson et al. (2012) considered the Mycobacterium avium subsp. Paratuberculosis (MAP) infection model by implementing environmental contamination along with Latin Hypercube Sampling and reweighted parameterization. Davidson and co-workers considered hypothetical estimation of herd related parameters. Robins et al. (2015) presented a model of MAP dynamics to assess the quarterly enzyme-linked immunosorbent assay (ELISA) and ethanol vortex ELISA (EVELISA) testing method in terms of economic analysis. In this model, the authors did not consider some important processes in a dairy herd like abortions for cows and heifers, still birth (calf that is either born dead or dies within the first 48 hours after birth), and milk yield variation. Recently, Al-Mamun et al. (2016) developed a dairy herd model where MAP infection dynamics were investigated, but this model also ignored some parameters like heifer pregnancy rate, milk yield, abortions, still birth, and milk yield variation.

To date, previous agent based models remain incomplete with respect to the normal herd operation setting. In reality, a farmer wants to know daily information about current milk production, stage of lactation, reproductive status, health, feeding practices, performance, costs and market prices from a computational model. Also, a farmer's ability to make the right decisions at the right times significantly determines the appropriate management of a progressive herd. Thus, we constructed a multiscale agent based simulation model of a dairy herd (MABSDairy), where we considered detailed herd operation processes and validated its performance with a dataset from real herds. We also improved our base model by including genetic potential and culling decisions coming from a pre-run DP model (Bar et al. 2008). First, we built the base dairy herd model by extending the model proposed in Al-Mamun et al. (2016) with new parameters. Second, five genetic potential parameters were added into the model to stratify milk yield by genetics. Third, a DP model proposed by Bar et al. (2008) was run to obtain culling parameters. Fourth, we assessed the performance of the proposed agent based simulation model (ABSM) without DP with DP culling parameters. Finally, a replacement policy was undertaken to demonstrate the application of the proposed ABSM.

The rest of the paper is organized as follows. Section 2 describes the method and parameters for building the base model and improved models. Section 3 presents the model setup, simulation results, relevant discussion for three models, and overall contribution of this work while presenting the possible applications and extension of the proposed model. Section 4 contains a summary of the model and concluding remarks.

2 THE DAIRY HERD MODEL

2.1 Overview of the Model

Our initial objective was to build a baseline model that would mimic the behavior and technical aspects of a normal herd operation over time. Therefore, a stochastic ABSM was created and implemented using MATLAB 2016a. We developed a base dairy herd model, then improved it further by incorporating stratification of genetic potential within cows and culling parameters from a pre-run DP model. Agents are defined as individual cows on a dairy herd. Each cow was tracked from birth to death. During this lifespan, each cow resides at three different management operations: adult/milking (aged >720 days), calf (aged 1-60 days) and heifer rearing housing (aged 61-719 days) (shown in Figure 1).

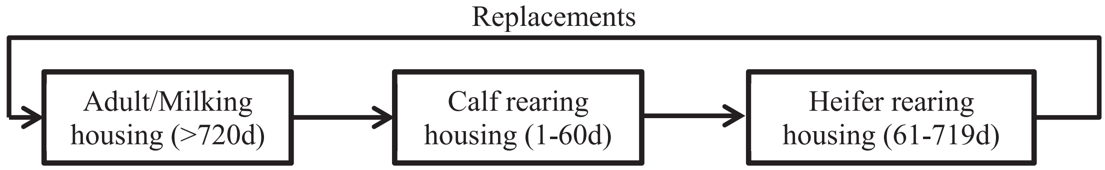


Figure 1: Management of operation cycle of a typical dairy herd. Abbreviation: d:days

2.2 Adult/Milking Herd

Adult cows must calve to produce milk. In a cow's adulthood, the lactation cycle refers to the period between one calving and the next. Generally, the lactation cycle is divided into four phases: voluntary waiting period (VWP), mid lactation, late lactation, and the dry off period (DOP). The VWP is observed in early lactation as a period of time in which adults are not inseminated even if they display estrus, to allow for optimum uterine involution and recovery from negative energy balance. If adult animals do not become pregnant within the optimal interval post-calving, due to unsuccessful insemination, the length of the lactation increases, but milk yield will gradually decrease during that extra time. Thus, achieving maximum milk production in a lactating herd depends on an optimal calving interval and culling open animals if they have not conceived even after repetitive inseminations. Our model included the generalized four-phase lactation cycle: VWP (up to 60 days after calving), insemination attempts (immediately after the VWP and every 21 days thereafter until pregnancy), pregnancy (280 days), and DOP (60 days prior to calving). During DOP farmers do not milk the cows. Figure 2 illustrates the gestation cycle of a dairy cow along with calfhood and heiferhood.

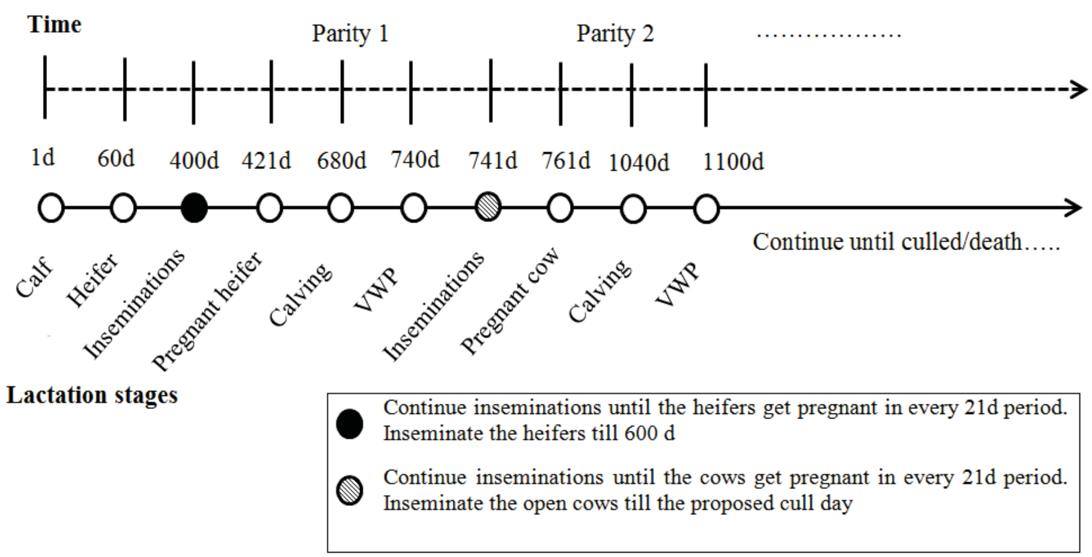


Figure 2: The lactation stages considered in the herd model. The bold circle indicates insemination attempts for making the heifer pregnant and the patterned circle indicates insemination attempts for making the cow pregnant. Abbreviations: d:days. VWP: voluntary waiting period

2.3 Herd Dynamics Flow

The model was initiated with a pre-determined distribution of animals with different parities. Every day, the algorithm first determined the group of animals. If it found adult animals, it checked reproductive status (VWP, waiting to be inseminated, and pregnant) and milk yield status (non DOP and DOP). Any cow on the 280th day of pregnancy was assumed to calve. For a newborn calf, the stillbirth probability was checked; if the calf was not stillborn, it was flagged as a calf. Only female calves were kept in the herd, and male calves were removed/sold immediately after birth. Once an adult animal calved, it transitioned to VWP status and continued in the milking herd loop until it was removed due to culling or death. Mortality was allowed in the calf rearing loop; otherwise, calves were transferred into the heifer loop at the 61st day of age. In the heifer loop, heifers were inseminated at the 400th day of age in order to become pregnant, so that they would calve at 680th days of age. When heifers were ready to calve for the first time, they transitioned to the milking herd in the model. Some sample pseudo code is displayed in Appendix A.

2.4 Reproduction and Culling

The reproductive process considered a cycle of events that included insemination, conception, pregnancy, abortion, and calving. For heifers, the first insemination attempt was set at 400 days. The model continued the inseminations every 21 days, until 605 days. If a heifer was not pregnant by 605 days, it was culled immediately. The standard probability of conception was 60% in heifers at first insemination, and decreased by 5% points with each subsequent insemination, with a minimum probability of conception of 45% for conventional insemination. For adult cows, the probability of conception was set to 35% for all inseminations (Galvão et al. 2013). Conception rate is calculated by dividing the number of cows confirmed pregnant by the total number of cows inseminated. Pregnancy rate was then calculated as the percentage of cows in a herd that become pregnant during every 21 days service period after the VWP.

The model considered two types of culling: overall culling and rebalance culling. Overall culling included voluntary and involuntary culling. Voluntary culling refers to culling for economic purposes (primarily milk production) and involuntary culling refers to culling due to natural death and accidents (De Vries et al. 2010). Rebalance culling was introduced to maintain approximately 200 ±10% of herd size. Based on rebalance culling, the surplus cows or heifers would be sold and if the number of adults dropped below 90% of the optimal size, purchase events would occur. Buying and selling occurred every month. In the base model, cows that were not pregnant by the 7th insemination after the previous calving were culled.

2.5 Milk Yield Model

Milk production for adult cows was included using an incomplete gamma function suggested by Wood (1967) throughout lactation. The incomplete gamma function is defined as

$$y_t = at^b e^{-ct} \tag{1}$$

where y_t is the yield on day t after calving, a is a scaling factor for initial yield, b is a rate factor for the increase in yield to peak, and c is a rate factor for the decline after peak. We used modified Wood (1967) lactation curves with parameters taken from Dematawewa et al. (2007) for parities 1, 2, and ≥3. The 305-d milk yields for these curves were 9,472; 10,819; and 11,136 kg for parities 1, 2, and ≥3, respectively.

2.6 Improved Model with Dynamic Programming

After building the base model, we improved the model using a DP model. The base model did not consider any genetic variation in cow categories. For improving our base model, we introduced pre-run DP parameters related to genetic potential and culling of open cows. The DP model was built using the multilevel hierarchic Markov process (MLHMP) software as the application program (Kristensen, 2003). The model was constructed as a 3-level hierarchic Markov process comprising the first (parent) level, which contained state variables of permanent traits throughout the cow's life span, the second level was divided into stages representing 1 whole lactation, and the third level was divided into stages of 1 month during lactation. We adopted a modified version of the DP from Bar et al. (2008). For further details on how a DP model works, see Bar et al. (2008). Our improved model is divided into two parts: improved model I (IM-I) and improved model II (IM-II). In IM-I, we stratified all the adult animals into five equal permanent milk yield ('genetic potential') categories, modeled as −5, −2.5, 0, +2.5, and +5 (kg) from the mean level of milk production per day. For day-to-day milk (temporary) production, we have added the relative milk yield based on transition probabilities. The transition between the monthly observed relative milk yield levels followed a normal distribution, with the mean equal to the permanent milk yield level and the variance estimated from the study farms. For example, the transition probabilities to the 5 temporary milk yield levels of +4, +2, 0, −2, and −4 kg of milk per day in the next month for a cow currently at a milk level of 0 were 0.07, 0.24, 0.38, 0.24, and 0.07, respectively; for a cow currently at level +4, the corresponding probabilities were 0.69, 0.24, 0.06, 0.01, and 0.00, respectively. In IM-II, we considered the days in milk (DIM) thresholds to cull the open cows. Table 1 presents the DIM when the model should cull open cows.

Table 1: Days in milk for when to cull the open animals supplied by pre-run Dynamic Programming (DP) model. 'Genetic Level' is expressed as deviations (kg) from mean level of milk production per day.

Parity	**Genetic Level (DIM to cull)**
1	5.0 (540d), 2.5 (480d), 0.0 (390d), -2.5(330d), -5.0(240d)
2, 3,4	5.0 (390d), 2.5 (360d), 0.0 (330d),-2.5(300d), -5.0(270d)
≥5	5.0 (390d), 2.5 (330d), 0.0 (300d),-2.5(270d), -5.0(240d)

Abbreviations: d:days, DIM: days in milk

2.7 Model Parameters

The commonly used parameters in three models are presented in Table 2. The parameters have been chosen from several published studies. Some model specific data have been extracted from the Regional Dairy Quality Management Alliance (RDQMA) (Pradhan et al. 2009) to validate the model's results.

Table 2: Basic parameters used during model construction.

Description	**Values**	**References**
Voluntary waiting period	60 d	Pradhan et al. (2009)*
Chance of producing female calf	0.5	USDA, (2007)
Gestation length	280 d	USDA, (2007)
Insemination period before culling for adults cow	225 d	Pradhan et al. (2009)*
Insemination period before culling for heifers	605 d	Kaniyamattam et al. (2016)
Daily probability of adult death/sold	0.00055	Assumed
Daily probability of calf death	0.000167	Assumed
Daily probability of heifer death	0.0000278	Assumed
Annual voluntary culling (%)	31	Pradhan et al. (2009)*
Adult pregnancy rate (%)	21	Pradhan et al., (2009)*
Stillbirth (%)	3	Meyer et al. (2001)
Daily risk of abortion for cow (%)	0.042	Kaniyamattam et al. (2016)
Daily risk of abortion for heifer (%)	0.029	Kaniyamattam et al. (2016)

Assumed: calibrated in the model, abbreviation: d:days

* Calculated from Regional Dairy Quality Management Alliance (RDQMA) dataset

Briefly, three commercial dairy farms (farm A in New York State, farm B in Pennsylvania, and farm C in Vermont) in the northeast United States were followed intensively for 10 years (2004–2014), with all health and culling events recorded. Apart from the common parameters, we considered modified daily culling parameters from Bar et al. (2008) and Al-Mamun et al. (2016) in IM-II. The new daily risk of culling of adult cows were 0.0002, 0.00033, 0.00067, 0.001, 0.0013, 0.0016, 0.002, and 0.0023 for the first, second, third, fourth, fifth, sixth, seventh, and eighth lactation, respectively.

2.8 Sensitivity

In order to improve the model's predictive capabilities and further development, a deeper understanding of how the variation of the output of the model can be apportioned, qualitatively or quantitatively, to different sources of variation must be investigated (Saltelli et al., 2000). We carried out a global sensitivity analysis of selected parameters, namely: cow conception rate (CCR), heifer conception rate (HCR), adult mortality (AM), calf mortality (CM), heifer mortality (HM), days to keep open heifers (DOH), and days to keep open cows (DOC). We explored the parameter space by performing an uncertainty analysis using the Latin hypercube sampling (LHS) method (Saltelli et al., 2000) conducted sensitivity analysis by evaluating partial rank correlation coefficients (PRCCs). Selected parameters were varied within 10% of their values.

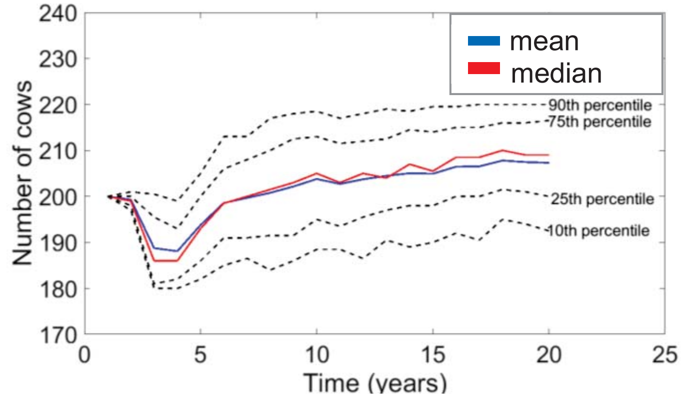
3 RESULTS AND DISCUSSION

3.1 Simulation Background

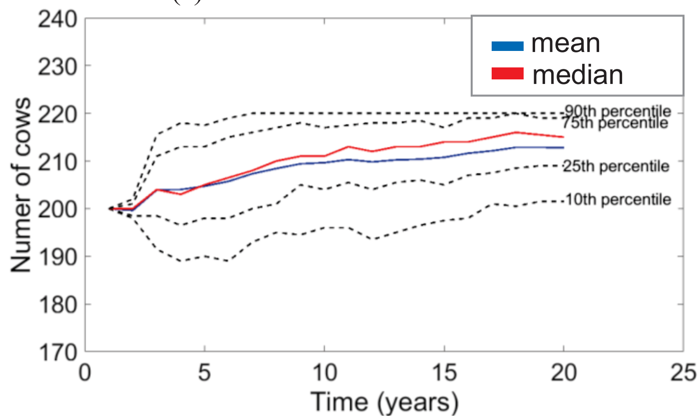
The model was initialized with 200 animals (as per USDA) with initial proportions of different parities (parity 1: 45%, parity 2: 25% and parity$\geq$ 3: 30%). For each parity, initial lactation parameters were determined. In the base model, genetic stratification was not included. IM-I included 5 different genetic potential and culling strategies suggested in the base model. In the base model and IM-I, any cows not pregnant by 7 inseminations were culled. In IM- II, however, culling parameters for open animals were chosen from Table 1. Also, modified voluntary culling parameters suggested in section 2.7 were considered in IM-II. The model was run for 22 years; the first 2 years were taken as 'burn in ' time. Each experiment consisted of 100 simulation runs.

3.2 Herd Statistics

Maintaining a herd requires information about basic factors like total cows, calves and heifers, conception rate, pregnancy rate, health, animal purchases, selling, etc. A simulation herd model must be validated with real herd parameters before implementing any management policy interventions. Figure 3 presents the number of available cows over a 20 years simulation period for the base model (Figure 3a) and IM-II (Figure 3b). The mean values suggest that both models maintain an adequate number of cows throughout the years.



(a) Number of cows in base model



(b) Number of cows in improved model II

Figure 3: Demonstration of number of cows every year, considering base model (a) and improved model II (b) in 100 simulation runs.

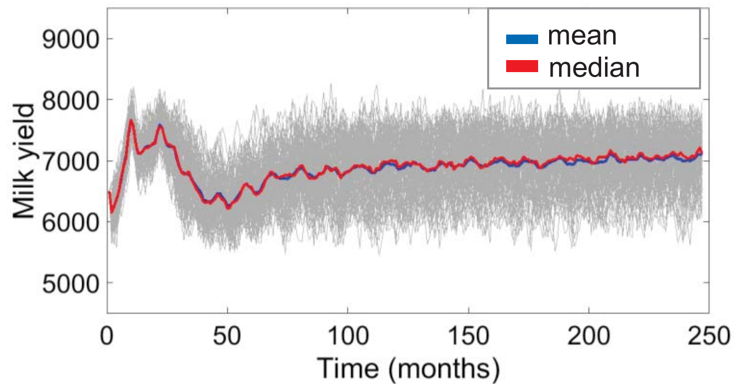
The essential parameters like replacements, calving interval, pregnancy rate and purchase and selling are presented in Table 3. It is seen from Table 3 that IM-II does not need any newly purchased cows, which indicates that using culling parameters from the pre-run DP corrected and improved our base model. Moreover, IM-II gives slightly more available replacements and extended calving interval.

Table 3: Statistical measurements of basic herd factors for base model and improved model-II. The values are presented as mean and 95% confidence interval (CI).

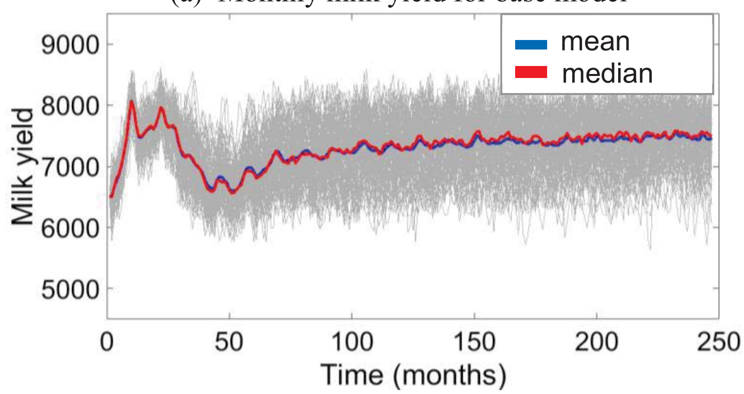
Herd factors	**Base model mean (95% CI)**	**IM- II mean (95% CI)**
Available replacements (calves + heifers)	131 (127-135)	139 (136-142)
Calving interval (days)	410 (400-420)	425 (416-434)
Pregnancy rate (per 21 days service)	21.5 (21.2-21.8)	18.1 (17.7-18.5)
Purchased cows	16.6 (15.6-17.6)	0 (0:0)
Sold cows	3.4 (2.6- 4.2)	3.2 (2.3-4.1)

3.3 Milk Yield

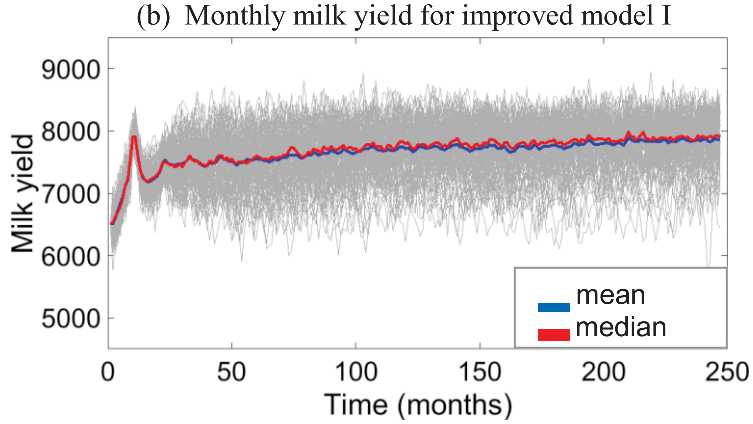
Figure 4 demonstrates the monthly milk yield curves for three models: base model, IM-I and IM-II. It is noticeable that introducing genetic potential into the herd corrected the monthly milk yield by 3%. It is also evident from Figure 4c that the farmer obtains the highest milk yield curve using IM-II. In the IM-II setup, the highest genetic potential cows were kept for a longer time to impregnate them. This is reflected in the milk yield curve.



(a) Monthly milk yield for base model



(b) Monthly milk yield for improved model I

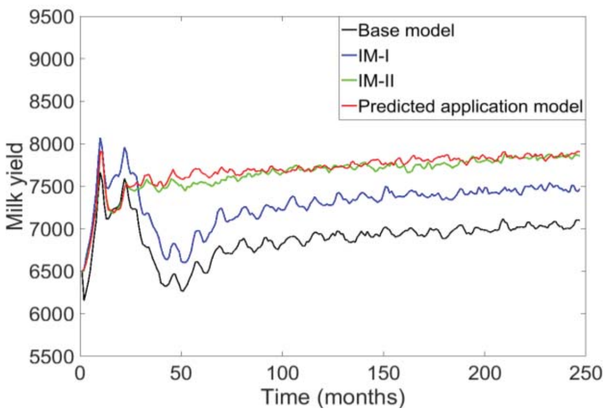


(c) Monthly milk yield for improved model II

Figure 4. Total monthly milk yield in 100 simulation runs for base model (a), improved model- I (b) and improved model- II (c).

3.4 Model Application

After simulating our improved model with genetic potential, we implemented an application scenario to assess its contribution in a real herd. We introduced a culling policy that not only depended on milk yield, but also depended on stratified genetic potential. It is generally assumed that a farmer will more likely want to keep a cow with high genetic potential rather than a cow with low genetic potential, even if their milk yield is temporarily low. In the long run, this policy may be more profitable than keeping a cow with low genetic potential having high temporal milk yield. To assess this, the farmer needs a decision making tool, which will inform him/her about the impact of milk yield while keeping cows with genetic high potential instead of culling them. First, the model ranked the cows based on their milk yield. Second, it predicted their 305 days milk yield based on genetic potential. Third, it ranked the predicted milk yield values. Fourth, based on that ranking it culled the low milk yield and low genetic potential cows. Figure 5a presents the predicted milk yield curve for the new culling policy. In 100 simulations runs, the average predicted milk yield was 7998.6± 206.8. The base model, IM-I and IM-II had average milk yields of 6890.9±237.5, 7647.7±202, and 7301.9±254.9 and, respectively. It is clearly evident that keeping cows with high genetic potential slightly increases the average monthly milk yield. In Figure 5b, an example of adopted culling strategy is shown, bold row indicates that the low genetic potential cows should be culled, as it stands at the lowest rank based on 305d predicted cumulative milk yield .



(a) Monthly predicted milk yeild

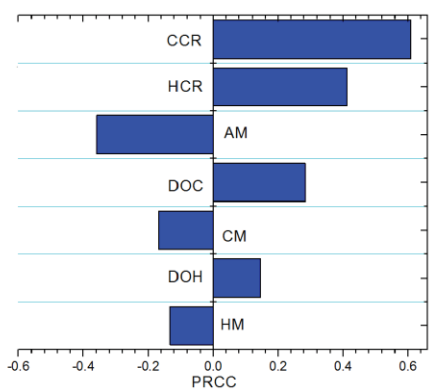
Genetic potential	Temporary MY	Rank	305d predicted MY	Rank
+5	32.4	3	11010	1
-5	**33.5**	**2**	**10132**	**3**
0	35.0	1	10302	2
…..	……	…..	…….	…..

(b) Culling list ranked by predicted cumulative milk yield

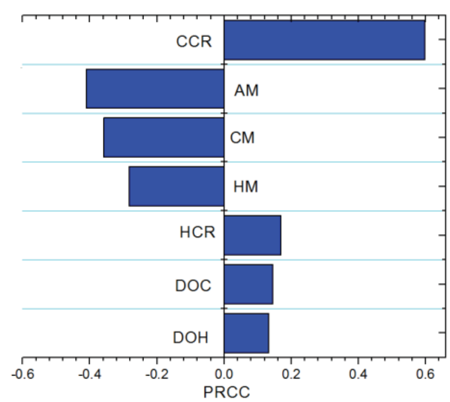
Figure 5: Monthly predicted milk yield curve (red line) after introducing a new culling policy based on genetic potential, temporary milk yield and cumulative predicted milk yield (a) and culling list, culled cow is marked as bold text (b). Abbreviations: d: days, MY: milk yield

3.5 Sensitivity

In Figure 6 selected input parameters are ranked by the partial correlation coefficient (PRCC) from highest to lowest by order for both models: base model and IM-II.



(a) PRCC for base model



(b) PRCC for improved model II

Figure 6. Sensitivity analysis of selected input parameters with respect to number of animals in the herd in the base model and improved model-II. Abbreviations: CCR: cow conception rate, HCR: heifer conception rate, AM: adults mortality, CM: calf mortality, HM: heifer mortality, DOH: days to keep open heifers and DOC: days to keep open cows.

Conception rate appears as the most important parameter for both models. In a real herd, conception rate for cows can vary within the range of 35-40% for conventional semen and the pregnancy rate can vary 15-25%. However, using sexed semen, farmers can achieve 80% conception probability for cows. Also, it is noticeable from Figure 6b that adult, calf and heifer mortality is negatively correlated with herd size.

4 CONCLUSION AND FUTURE WORKS

The objective of this paper was to build an ABSM for a normal dairy herd operation where we considered accurate parametrization from published literature and an RDQMA dataset (Pradhan et al. 2009). To fulfill the objective, we built a base model with accurate parameterization and later improved into two versions: IM-I and IM-II. The simulated results showed significant resemblance to real herd dynamics parameters. For planning and managing a farm operation, a farmer requires information from different layers: individual cow level, population level and management and policy level. Sections 3.2 and 3.3 present the population level information like number of total animals (Figure 3), available replacements and other factors (Table 3). Then, we tested the model with a simple application and introduced a new monthly culling policy based on genetic potential and milk yield. Simulation results showed that predicted future 305 cumulative days milk yield improved the average monthly milk yield (shown in Figure 5).

Our existing model has some limitations, as information and knowledge are always incomplete (Delorenzo and Thomas, 1996). The current version ignored some economic parameters for cows, such as body weight, dry matter intake, feed cost, somatic cell count, milk protein and fat, management related costs, etc. However, the current model is adaptive in nature, so such parameters can be readily incorporated in the future. This model is capable of providing detailed information about an individual animal's daily life with different discrete events. This kind of stochastic agent based model is suitable for investigating contagious diseases like foot and mouth disease (FMD), bovine TB (bTB), mastitis, and Johne's disease (JD) in cattle herds. This model also allows economic modelers to collect information about events for each cow and penalties for each disease state. Beyond these uses, this model can be thought of as a framework for similar dynamic systems where different layers of information contribute in decision making. In the future, our aim will be to add economic parameters to the model to make it more usable in a real dairy herd.

Overall, this paper presents an ABSM of a dairy herd named MABSDairy. The MABSDairy was built with accurate parameterization and validated against an existing dataset. The results from this model offer valuable information to farmers to predict optimal management decisions. Also, this model offers an adaptive framework which can be used by the users not only to investigate disease related problems, but also to calculate economic values of optimal management decisions in cattle herds.

ACKNOWLEDGMENTS

This project was supported by Agriculture and Food Research Initiative Competitive Grant no. 2014-67015-2240 from the United States Department of Agriculture through National Institute of Food and Agriculture.

A APPENDIX

Pseudo code for herd dynamics

```
Algorithm: herd dynamics
Precondition: Initial states of the model are set with 200 agents with random age and
different parities
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```
while there are any adults, agents do:
check the status
 if voluntary waiting period, then
    wait 60 days; increase age by 1
 elseif inseminations
    check the probability of insemination
    if insemination is successful, then
       flagged as pregnant
    else
       wait for next day
    end if
    elseif pregnant
       check the pregnancy date (preg_d) == 280
       if preg_d == 280, then
          calving and stay with mother for one day
          pick a milk yield value from woods curve equation 1
       end if
       else
          increase age by 1
       end if
    end if
end while
while there are any calves, agents do: {…..
                                            … }
end while
while there are any heifers, agents do: {…..
                                            … }
end while
```

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