The Hermite-Taylor Correction Function Method for Maxwell's Equations

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Hermite-Taylor methods are high-order methods, especially well-suited for hyperbolic problems, that rely on a Hermite interpolation procedure in space and a Taylor method to evolve the data in time. These methods are efficient since the time-step size needs to satisfy a stability condition that depends only on the maximum wave-speed and is independent of the order [2]. However, the enforcement of general boundary conditions is still a challenge since Hermite-Taylor methods require not only to know the electromagnetic fields on the boundary but also their m first derivatives in space to achieve a $(2\,m+1)$-order method. Chen et al. [1] proposed a hybrid DG-Hermite approach to circumvent this issue by taking advantage of a DG solver in the vicinity of the boundary but a local time-stepping procedure is required to retain large time-step sizes in Hermite methods.

In this talk, we propose another avenue based on the Correction Function Method (CFM) [3, 4] to seek all the needed information on the boundary. The CFM relies on the minimization of a functional that is a square measure of the residual associated with Maxwell's equations. This minimization problem has to be solved for only some nodes in the vicinity of the boundary at each time step and some pre-computations can be done to reduce the computational burden of the CFM. The CFM has been applied to FDTD schemes and the resulting schemes achieve up to fourth-order convergence while considering complex embedded boundaries. Here, we investigate the CFM in the Hermite-Taylor method setting. Numerical examples in 1-D and 2-D are performed and the expected convergence order is observed for reasonable values of m.

References

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