The Active Flux Method: Is Superconvergence Possible?

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The Active Flux method was developed by Eymann and Roe [2] for linear hyperbolic partial differential equations. The Active Flux method is a third order accurate method that constructs a continuous quadratic polynomial by evolving the cell average and element interface values, where the interface values are calculated using information from the characteristics. For advective transport, there is a straight forward extension of the Active Flux method to cartesian grids containing cut cells that is stable under a CFL condition for the regular cartesian grid. No further stabilisation is necessary [3]. Borrowing ideas from (discontinuous Galerkin) finite element methods, we will investigate whether superconvergence of the Active Flux approximation is possible and, if so, how to go about extracting this extra information for an increased convergence rate.

The specific method that we explore is the Smoothness-Increasing Accuracy-Conserving (SIAC) filter. This filter is normally utilized as a post-processor that extracts accuracy from (DG)FEM solutions by exploiting the natural superconvergence of these methods. It relies on the error in the divided differences being the same order as that for the numerical scheme. For these types of approximations, it reduces oscillations in the errors and increases convergence rates, leading in general, to lower errors. In order to extend these ideas to Active Flux methods, we explore the framework needed to ensure an increased convergence rate and the advantages and disadvantages of utilizing the SIAC filter in one- and two-dimensions.

References

[1] J. Docampo-Sánchez, G.B. Jacobs, X. Li and J.K. Ryan. *Enhancing accuracy with a convolution filter: What works and why!* Computers & Fluids, vol. 213, 2017

[2] T.A. Eymann and P.L. Roe. *Multidimensional Active Flux Schemes.* AIAA Conference Paper, June 2013.

[3] David C. Kerkmann.Active. *Flux Methods for Conservation Laws on Complex Geometries.* PhD thesis, Heinrich-Heine University DÃ¼sseldorf, 2021