Stable Discretizations of Spectrally Convergent Radiation Boundary Conditions

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Abstract

As the radiation of energy to the far field is a central feature of wave physics, convergent near-field domain truncation techniques are an essential component of any comprehensive software for simulating waves, particularly any which wish to leverage the advantages of high-order volume discetizations. Many methods have been proposed to address this issue. Focusing on time-domain formulations, examples include retarded potential integral equations [1], fast low-memory methods for applying space-time integral operators arising in exact formulations [2, 3], damping layers such as the perfectly matched layer [4] or simpler combinations of grid stretching and artificial viscosity [5], as well as sequences of local radiation boundary conditions. Here we advocate the latter technique. First, in comparison with methods based on integral operators, local methods are more easily parallelized and are more memory-efficient. Second, in comparison with damping layers, there is a straightforward way to choose optimized parameters associated with sharp error estimates. These are the complete radiation boundary conditions (CRBC) [6]. Precisely, CRBC is implemented via the evolution of P auxiliary variables, ϕ_j, on or near the radiation boundary. A sharp *a priori* error estimate shows that an error tolerance ϵ can be guaranteed up to time T with

$$P \approx \ln\left(\frac{cT}{\delta}\right) \cdot \ln\left(\frac{1}{\epsilon}\right).$$

Here c is the wave speed and δ is the separation of wave sources/scatterers from the radiation boundary.

The evolution problems for the auxiliary variables ϕ_j differ depending on the structure of the governing equations. For first order hyperbolic systems, these auxiliary functions may be restricted to the faces of the radiation boundary [7]. For second order systems they can be evolved in a thin double absorbing boundary layer [8, 9]. In both cases edge and corner closures can be imposed to support polygonal artificial boundaries. Although the evolution equations for the auxiliary equations are provably hyperbolic, they do not seem to be symmetrizable using local inner products and thus we have not been able to derive simple energy estimates. Therefore, standard techniques for proving the stability of Galerkin discretizations do not directly apply. Instead, methods based on Laplace transforms are employed. It is worth noting that such techniques are needed even to analyze the exact

radiation conditions, as the energy in bounded subregions is generally not monotonically decreasing.

Here we present theoretical and experimental studies of stable, high-order discretizations of the auxiliary equations as well as a discussion of their coupling with popular volume discretizations. These are the necessary ingredients for our development of **rbcpack** [10], a generally usable CRBC library.

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