LOW-ORDER TOOLS FOR HIGH-ORDER FINITE ELEMENTS

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High-order numerical methods promise numerous benefits over traditional low-order methods: higher accuracy per degree of freedom, more predictive results for under-resolved simulations, and better resolution of complex, curved geometries, among others. Additionally, performance on modern supercomputing architectures is of increasing importance, and high-order methods possess favorable features, such as high arithmetic intensity and structured memory access, making them amenable to high-performance implementations on GPU-based architectures.

However, despite these advantageous properties, the use of high-order methods also presents some challenges. For example, the linear systems arising from high-order discretizations are typically ill-conditioned. These systems become increasingly dense, and often cannot be stored in GPU memory, ruling out the use of traditional preconditioning techniques such as algebraic multigrid, and necessitating the development of matrix-free preconditioners. Futhermore, the application of high-order methods to advection-dominated problems such as systems of hyperbolic conservation laws, and their use together with low-order methods in the context of multiphysics simulations introduce new challenges, including monotonicity, invariant domain preservation, and conservative solution transfer.

In this talk, I will describe several applications of a *low-order-refined* methodology, with the goal of addressing some of these challenges. This technique is based on the idea of forming an auxiliary lowest-order discretization on a refined mesh, and using favorable properties of the low-order method to augment or improve the high-order discretization. Classically, this technique has been used to construct preconditioners for the Poisson problem (cf. SEM–FEM equivalence, introduced by Orszag in 1980); in this talk, I will discuss the systematic construction of low-order-refined preconditioners for the entire finite element de Rham complex, including large-scale problems in $H(\mathrm{curl})$ and $H(\mathrm{div})$, featuring optimal computational complexity and memory requirements, and uniform convergence, independent of mesh size and polynomial degree. These preconditioners make use of a specially constructed interpolation–histopolation basis, and both the preconditioner setup and its application are amenable to GPU-acceleration. Additional applications of the low-order-refined methodology include the construction of invariant-domain-preserving methods for hyperbolic conservation laws, and the accurate and conservative solution transfer between high-order and low-order representations.