LSIAC-MRA: Enhanced Multi-Resolution Analysis via Line SIAC Post-Processors

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Abstract

Line Smoothness-Increasing Accuracy-Conserving Multi-Resolution Analysis (LSIAC-MRA) is a procedure for obtaining more accurate multi-resolution analysis by application of convolution kernel post-processors [1]. Leveraging SIAC filters and projection techniques, LSIAC-MRA allows for the transition of data from coarse to fine resolutions while simultaneously decreasing errors in the fine grid approximation as depicted in Figure 1. This construction of fine grid information from coarse grid data allows for the computation of multi-wavelet coefficients which would otherwise not be defined for coarse grid data. The procedure consists of a filtering stage in which the coarse data is post-processed. This is followed by a projection stage in which the filtered approximation is projected onto a finer mesh. Accounting for the construction of the convolution kernel and mesh geometries, we evaluate exactly the resulting convolutions. This leads to a filtered approximation expressible in a new basis. This basis expansion form for the filtered approximation allows for discretization of this stage as a simple matrix multiplication. Fine grid modes are then obtained by projection of the filtered basis onto the fine grid basis. Though presented in the context of the scaling functions and multi-wavelets of Alpert [2], this procedure is applicable to other data types and choices of basis function. We demonstrate the effectiveness of this technique for error reduction in multi-dimensions.

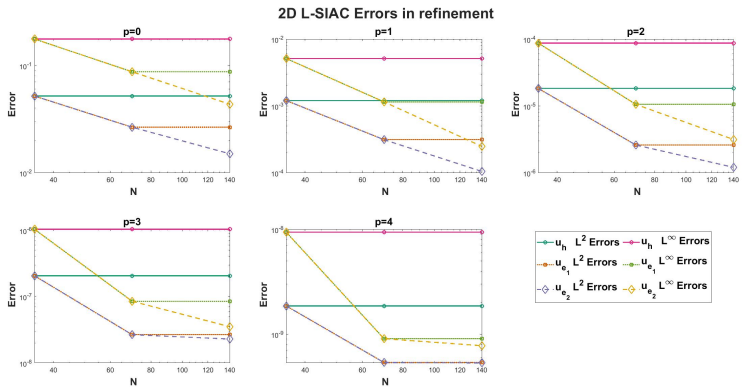

Figure 1: Log-log error plots for enhanced approximations of the L^2-projection of $u(x,y) = \sin(2\pi(x+y))$ initialized on a 35×35 mesh of $[0,1]^2$.

References

[1] M.J. Picklo and J.K. Ryan. *Enhanced Multi-Resolution Analysis for Multi-Dimensional Data Utilizing Line Filtering Techniques.* arXiv 2108.05769v2 (2021).

[2] B. K. Alpert. *A Class of Bases in L^2 for the Sparse Representation of Integral Operators.* SIAM J. Math. Anal. (1993),