INCREASING COMPLEXITY VEHICLE MODELS FOR ANALYZING AND DESIGNING AUTOMOTIVE CONTROL SYSTEMS

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ABSTRACT

Bond Graphs provide a systematic approach to modeling physical systems of varying complexity and energetic systems/subsystems. In automotive engineering research, this modeling method is suitable such that it becomes possible to coincide the conceptual design stage with a vehicle analysis model of appropriate detail. The resulting high fidelity models can reasonably predict performance of the proposed control or design for a vehicle subsystem, while continuing to provide analytical findings and physical insight. This paper demonstrates the incremental formulation of such vehicle models for use along any stage of the automotive research process relating to ride and handling enhancements. The models presented will begin with a commonly referenced bicycle model and progress to an extensive model that captures multiple vehicle body degrees-of-freedom along with increasing details for subsystems such as steering, braking, drivetrain, and more.

Keywords: vehicle dynamics, system modeling, automotive engineering.

1 INTRODUCTION

Automatic control is prevalent in numerous automotive engineering systems. All segments of vehicles are continuously evolving to suit the growing need of consumers while placing consideration on alternative technologies to reduce power consumption for the future. The movement toward electrification and driver automation highlights the need for large scale reconfiguration of new vehicle component systems and integration with the vehicle. Even changes within a single vehicle component requires thorough analysis and control considerations to ensure compatibility with the other vehicle subsystems.

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One key item is required for both concept generation and development of integrated control frameworks: a suitable model. Such a model provides the necessary insight of the physical nature of the system analytically while generating predictive results through simulation testing. It should be understood that the modeler decides which dynamic behavior is of interest and includes them accordingly. A well informed control system is then able to provide an appropriate control action that is based on the predictions and intuition obtained from the model. Additionally, it is important to note that while the development of a component evolves, the model must also follow. The model synthesis proposed here reflects this incremental progression. A high level conceptual idea is matched with a simpler model and as the concept matures so does the complexity of the model.

The bond graph modeling method has been employed for many years in vehicle dynamics research. It is a graphical approach to modeling any multi-energy physical system (mechanical, electrical, hydraulic, etc.) which streamlines the modeling process for various vehicle components. Bond graphs are also advantageous for modeling subsystems which can be assembled into an integrated system. This characteristic is possible through the assignment of causality, or input-output relationship. In vehicle systems the input-output relationship between components remain consistent when component changes are applied, allowing for the flexibility of analyzing individual systems of the vehicle without modifying the remainder of the model.

In the following sections a general procedure for modeling the integration of vehicle subsystems is presented. The focus will mainly be on planar dynamics as it relates to the handling qualities of the vehicle. A fundamental vehicle model is modified with increasing complexity and with the addition of subsequent subsystems. First-order equations of motion result from the bond graph using a prescribed procedure lending itself useful for simulation and control development. Documentation for the bond graph method and procedures can be referenced in (Karnopp, Margolis, and Rosenberg 2012).

2 SYSTEM MODEL

The vehicle system consists of the following: chassis body, suspension, steering, drivetrain, and powertrain. All of components interacting with the chassis body can be modeled as detailed subsystems or simply inputs (effort or flow) depending on the scope of study. These sections will demonstrate incremental process of adding complexities to a vehicle model. Utilizing fixed causality relationships will enable the addition, removal, or modification of the model without having to change the remainder of the system model. To refresh for those familiar with bond graphs and to inform those who are not, causality is a notation on a bond graph that dictates the effort and flow exchange between two systems. A causal mark on a bond defines the input and output relationships between systems and a system may only have one configuration: 1) effort in and flow out or 2) flow in and effort out. This means the adjacent system has the inverse configuration. By maintaining causality, modular analysis of systems are possible without affecting the equations of motion of the rest of the system. The following sections will describe the build up of the models. See Table 1 for an outline of the models and their descriptions.

In general, the vehicle is modeled as a 3-D rigid body with body fixed coordinates centered at the cg and oriented along principal directions. These principal velocities are the outputs of the chassis body while the inputs are the forces and moments that act on the body. Intermediate velocities at different points along the chassis body can be assembled with the multiport elements: zero junction and one junction. This input-output relationship will be held constant for the modeling procedures carried out. The diagram in Figure 1 describes the interactions between the body and the rest of the vehicle subsystems.

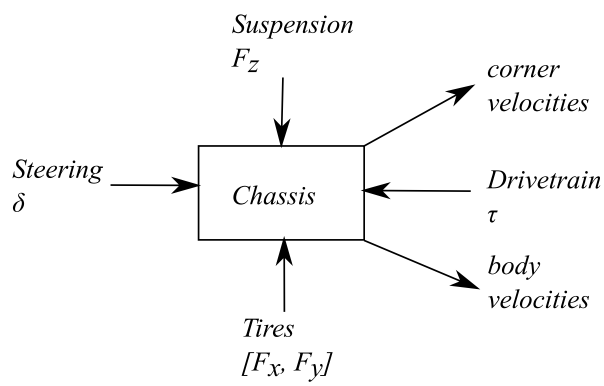

Figure 1: Vehicle Chassis Body Input/Output Relationships

2.1 Bicycle Model

The bicycle model is chosen as the fundamental model as it is widely used for analyzing the planar vehicle dynamics. This model has front wheel steering and is assumed to be given a prescribed forward velocity. Figure 2 shows its defined translational forward and sideways velocites as u and v while the angular velocity in the yaw direction is ω. The vehicle has mass m, yaw moment of inertia J, cg to front axle distance a, and cg to rear axle

Table 1: Summary of Vehicle Model Build Up

Model	Addition	Parameters	Inputs
Bicycle Model (FWS)		$m\ J\ a\ b\ C_f\ C_r$	δ_f
Bicycle Model (F/RWS)	rear wheel steer		δ_r
	caster effects	$C_{a_f}\ C_{a_r}\ J_w$	
	bushing effects	$m_w\ k_f\ k_r$	
Extended Bicycle Model (4WS)		$m\ J\ a\ b\ C_f\ C_r$	$\delta_{lf}\ \delta_{rf}\ \delta_{lr}\ \delta_{rr}$
		$m_w\ J_w\ R_w$	$\tau_{lf}\ \tau_{rf}\ \tau_{lr}\ \tau_{rr}$
		$h_g\ w\ k_{\tau_f}\ k_{\tau_r}$	
	drivetrain detail	$f_{ac}\ g_r$	$\tau_e\ \tau_{B_{lf}}\ \tau_{B_{rf}}\ \tau_{B_{lr}}\ \tau_{B_{rr}}$

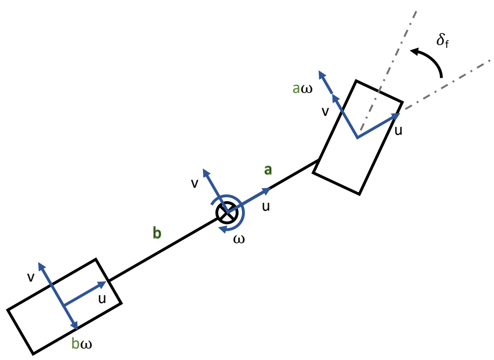

Figure 2: Bicycle Model (FWS)

distance b. Inputs to the vehicle are steering angle δ_f and tire forces F_{y_f}, F_{y_r}.

The tire forces are generated through slip angles which are defined as a ratio of the wheel sideways velocity to its forward velocity (Eq.1). The vehicle cg velocity components are transferred to the wheels yielding the slip angles. One approximation of tire forces is a linear relationship with the slip angle through cornering coefficients C_f and C_r. These coefficients take into account the total effect of both tires along each axle, hence, the 'bicycle' name. The resulting tire forces are $F_f = C_f\alpha_f$ and $F_r = C_r\alpha_r$. Equation 2 is an equivalent equation but modified to represent a constitutive law of a linear resistance element.

Slip angles:

$$
\begin{aligned}
\alpha_f &= \delta_f - \frac{v + a\omega}{u} \\
\alpha_r &= \frac{b\omega - v}{u}
\end{aligned}
\tag{1}
$$

Tire Forces:

$$
\begin{aligned}
F_f &= \frac{C_f}{u}\Big(\delta_f u - (v + a\omega)\Big) \\
F_f &= \frac{C_r}{u}(b\omega - v)
\end{aligned}
\tag{2}
$$

Figure 3 shows a bond graph of the bicycle model with linear tires. Inertia elements in integral causality are attached to each body fixed velocity. This is consistent with the idea of flow output from the body and effort input into the body. The tires are modeled as resistive elements applying an effort into the system based on the kinematic relationship at the wheels. When using body fixed coordinates, modulated gyrators are required to produce the cross-product terms that emerge from Euler's equations (see (Karnopp 1976)). A modulated transformer is also introduced for the $\delta_f u$ term of the front wheel slip angle.

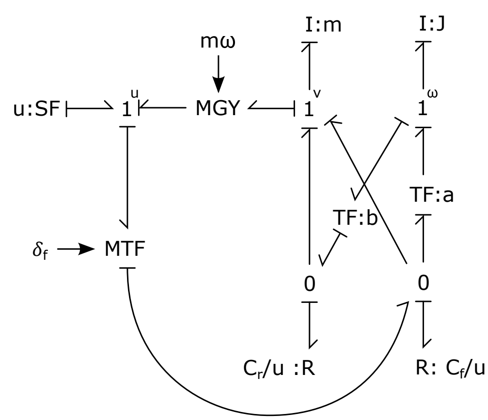

Figure 3: Bicycle Model (FWS)

2.2 Bicycle Model - RWS

The addition of rear wheel steer to the bicycle model simply introduces a steering angle input to the rear wheel. The slip angle for the rear adds a new term δ_r. The bond graph changes with the introduction of another modulated transformer that connects the forward velocity u to the zero junction of the resistive element for the rear tire.

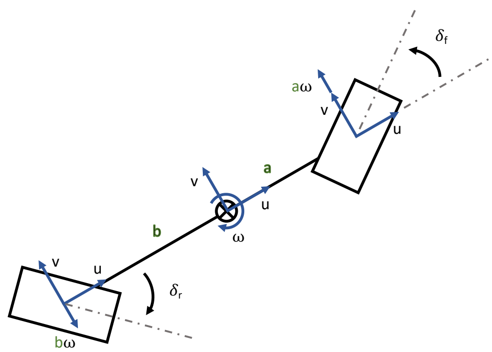

Figure 4: Bicycle Model with Rear Wheel Steer

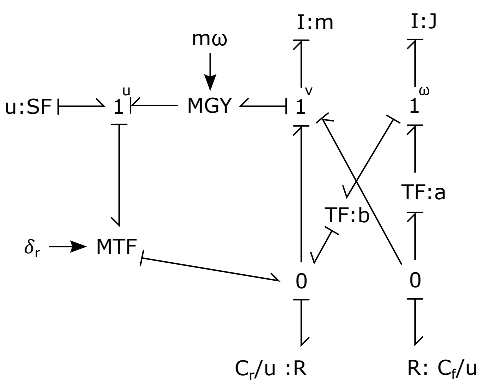

Figure 5: Bond Graph of Bicycle Model with Rear Wheel Steer

2.3 Bicycle Model (RWS) - Caster

The steered wheels of a vehicle are positioned with a caster angle (the angle between the steering axis of the wheel and the vertical axis). The caster effect provides a stabilizing effect on the vehicle in the absence of a steering torque by straightening out the wheel due to the caster trail moment arm and the lateral force from the ground. Figure 6 shows a diagram of a vehicle with caster effects. The bond graph of the bicycle model with caster effect has additional inertias to account for the rotation of each wheel about its diametral axis (an approximation about the steering axis). The trail distance is a moment arm about which the tire force acts on, therefore, a transformer element is applied to the bond graph between the rotational velocity of the wheel ω_w and the zero junction to the resistive element representing each tire.

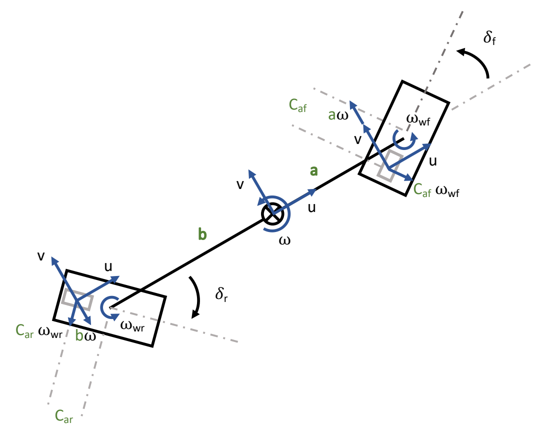

Figure 6: Bicycle Model with Caster

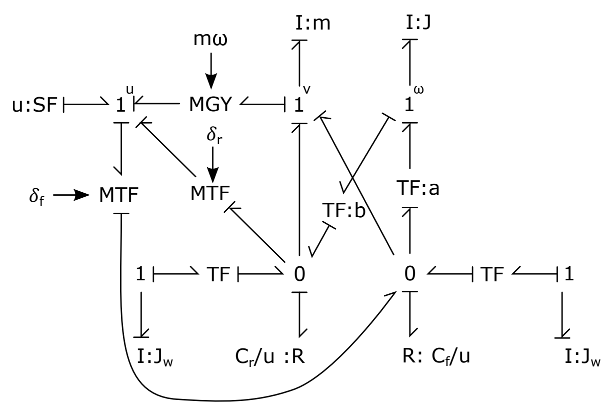

Figure 7: Bond Graph of Bicycle Model with Caster

2.4 Bicycle Model (RWS) - Caster and Bushing

Bushings are introduced in vehicle joints and attachment point to address noise, vibration, and harshness concerns. This model sheds light on the lateral component of the wheel attachment bushing. The other components of the bushing (longitudinal and torsional) are omitted here but can be added to this model. The lateral component of the bushing is modeled as spring that is modulated by the relative velocity between the chassis and the wheel. As a result, the wheels are considered as rigid bodies with their own body fixed coordinates with mass m_w and diametral moment of inertia J_w. The velocites of front wheels are u_f, v_f, and ω_{w_f} while the velocities of the rear wheels are u_r, v_r, and ω_{w_r}. The model will be simplified by allowing $u_f = u_r = u$ due to small angle approximations of the wheel relative to the vehicle body. As a result the bond graph has the following additional elements: inertia for each wheel in the lateral direction, compliance and resistive elements for the bushing, and modulated gyrators coupling the wheel forward velocity (same as chassis body forward velocity) to the wheel lateral velocity.

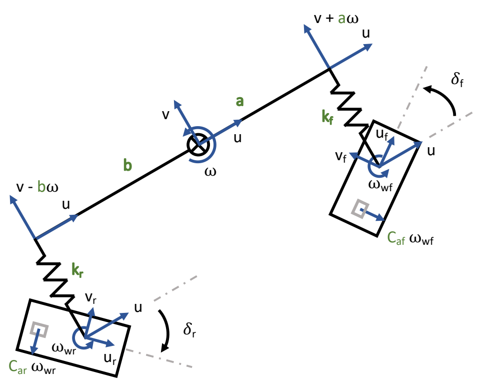

Figure 8: Bicycle Model with Caster and Bushing

2.5 Extended Bicycle Model

An extended bicycle model is an extension of the bicycle model which has only planar vehicle dynamics. The model has four wheels with independent steering control and independent wheel torque (brake and traction) control. Limit handling performance is typically the motivation for using this model. In such cases, the tire-road friction limit is approached during simultaneous steering and brake/throttle input, thus, more sophisticated tire models are needed for properly analysing this

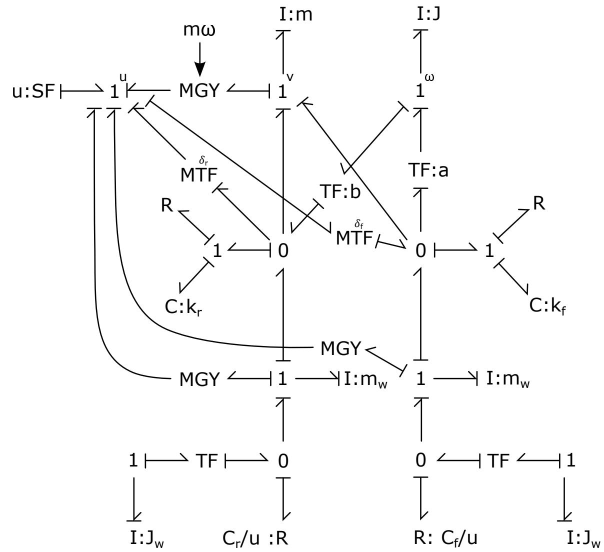

Figure 9: Bond Graph of Bicycle Model with Caster and Bushing

system. The Dugoff tire model is selected and is discussed later in this section.

The extended bicycle model still has the same body fixed velocities u, v, and ω and as a result the same inertia properties m and J. The velocities of the wheels are now about the rotation axis of the spinning wheels (the bicycle model was previously concerned about the diametral axis of the wheels) with mass properties m_w and J_w. The steering angle input is extended to four individual steering inputs (δ_{lf}, δ_{rf}, δ_{lr}, and δ_{rr}) while the torque inputs at each wheel are (τ_{lf}, τ_{rf}, τ_{lr}, and τ_{rr}). Additional parameters include cg to axle distances a and b, cg height h_g, half track distance $\frac{w}{2}$, wheel radius R_w, and front and rear roll stiffness k_{τ_f} and k_{τ_r}.

In contrast to the bicycle model with linear tires, the extended bicycle model requires a nonlinear tire model that captures realistic tire behavior at the friction limit of tire-road interaction both in the lateral F_y and longitudinal directions F_x. The Dugoff tire is an example of a tire model that can provide such information at the cost of only a few additional vehicle states including: slip s, slip angle α, and normal force N (Dugoff, Fancher, and Segel 1970). The normal force is introduced because changes in lateral and longitudinal acceleration causes changes in tire loadings. This normal force distribution, which is also influenced by roll

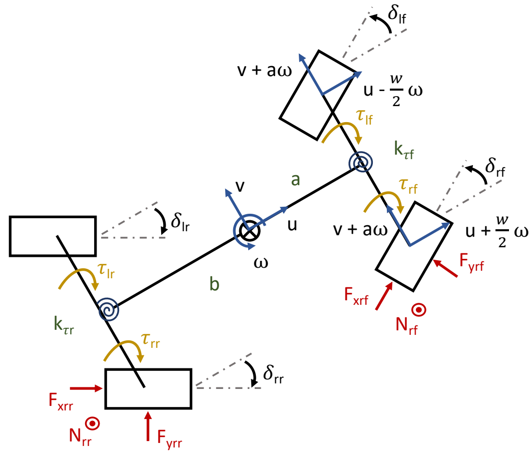

Figure 10: Extended Bicycle Model

stiffnesses, produces effects on the generated lateral and longitudinal tire forces. The definitions of slip, slip angle, and algebraic normal force are presented below.

Longitudinal Slip (s):

$$
\begin{aligned}
s_{lf} &= \frac{R_w\omega_{lf} - \left(u - \frac{w}{2}\omega\right)}{|R_w\omega_{lf}|} \\
s_{rf} &= \frac{R_w\omega_{rf} - \left(u + \frac{w}{2}\omega\right)}{|R_w\omega_{rf}|} \\
s_{lr} &= \frac{R_w\omega_{lr} - \left(u - \frac{w}{2}\omega\right)}{|R_w\omega_{lr}|} \\
s_{rr} &= \frac{R_w\omega_{rr} - \left(u + \frac{w}{2}\omega\right)}{|R_w\omega_{rr}|}
\end{aligned}
\tag{3}
$$

Lateral Slip Angle (α):

$$
\begin{aligned}
\alpha_{lf} &= \delta_{lf} - \frac{v + a\omega}{u - \frac{w}{2}\omega} \\
\alpha_{rf} &= \delta_{rf} - \frac{v + a\omega}{u + \frac{w}{2}\omega} \\
\alpha_{lr} &= \frac{b\omega - v}{u - \frac{w}{2}\omega} - \delta_{lr} \\
\alpha_{rr} &= \frac{b\omega - v}{u + \frac{w}{2}\omega} - \delta_{rr}
\end{aligned}
\tag{4}
$$

Algebraic Normal Force (N):

$$
\begin{aligned}
N_{lf} &= \frac{1}{2}m\left(\frac{bg - h_g a_{long}}{a+b} - a_{lat}\frac{h_g}{w/2}\frac{k_{\tau_f}}{k_{\tau_f} + k_{\tau_r}}\right) \\
N_{rf} &= \frac{1}{2}m\left(\frac{bg - h_g a_{long}}{a+b} + a_{lat}\frac{h_g}{w/2}\frac{k_{\tau_f}}{k_{\tau_f} + k_{\tau_r}}\right) \\
N_{lr} &= \frac{1}{2}m\left(\frac{ag + h_g a_{long}}{a+b} - a_{lat}\frac{h_g}{w/2}\frac{k_{\tau_r}}{k_{\tau_f} + k_{\tau_r}}\right) \\
N_{rr} &= \frac{1}{2}m\left(\frac{ag + h_g a_{long}}{a+b} + a_{lat}\frac{h_g}{w/2}\frac{k_{\tau_r}}{k_{\tau_f} + k_{\tau_r}}\right)
\end{aligned}
\tag{5}
$$

2.6 Extended Bicycle Model - Drivetrain

An alternative drivetrain configuration is developed for the extended bicycle model. Instead of individual torques applied at the wheel (as is the case for in-wheel motors) the more conventional approach is to model an engine torque that then gets distributed to the wheel. The specified engine torque τ_e is tranferred to the front and rear of the vehicle through a center differential by a factor f_{ac}. The axle torque is further transformed into individual left and right driveshaft torques through a limited slip differential resulting in all four individual traction torques (τ_{lf}, τ_{rf}, τ_{lr}, and τ_{rr}). In addition to the traction torque, a separate brake torque is applied to each wheel ($\tau_{b_{lf}}$, $\tau_{b_{rf}}$, $\tau_{b_{lr}}$, and $\tau_{b_{rr}}$). All of these variables provide the ability to test strategies of integrated chassis control relating to the differentials, brake, and steering. Note: the bond graph presented here will be a front wheel steer configuration (rws can be easily integrated).

3 CONCLUSION

The bond graph modeling approach has been demonstrated as a tool to develop multisystem vehicle dynamic system models. The system models are created with thoughtful detail yet simple enough to trace specific dynamic behavior of the system. Equations of motion are readily available in first order form lending itself useful for computer simulation. Causality and power bonds are used to relate multi-energy systems making bond graphs suitable for automotive research and development. As long as causality is preserved, different subsystems can be introduced to the model. Creat-

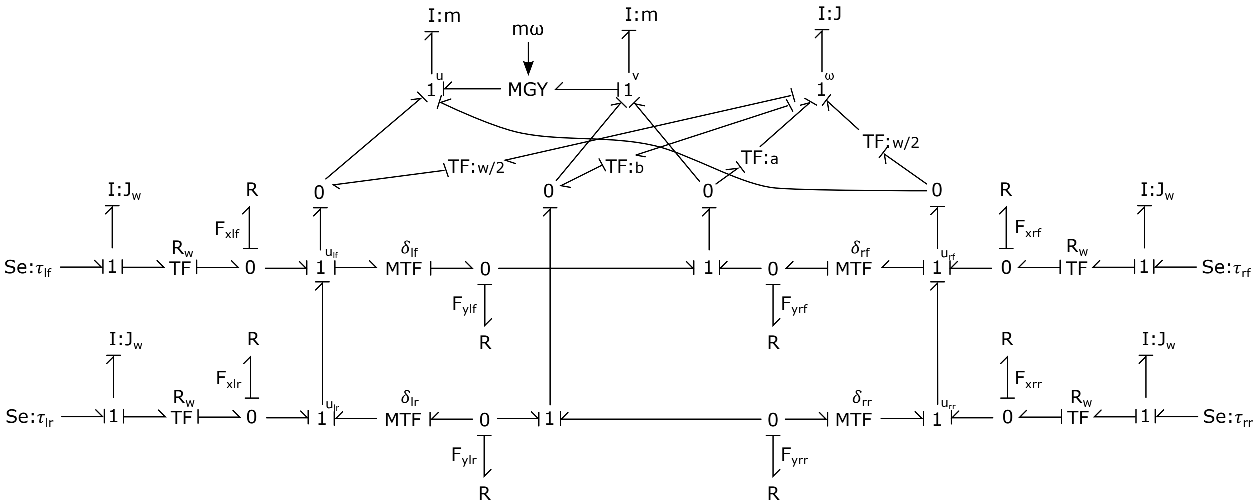

Figure 11: Bond Graph of Extended Bicycle Model

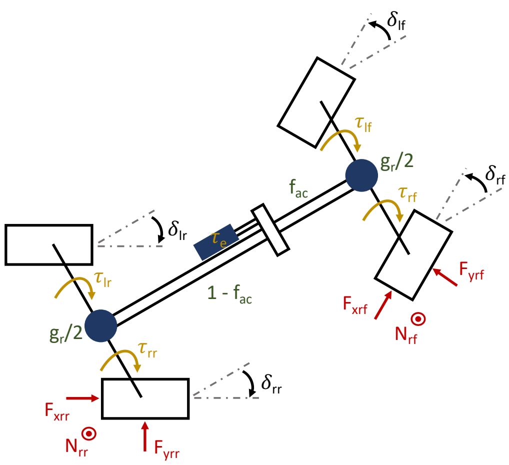

Figure 12: Extended Bicycle Model with Drive-train

ing vehicle dynamics models and using the bond graph method such as the examples presented here will provide time and cost savings. These models can be integrated within any stage of virtual simulation and control providing data for performance analysis.

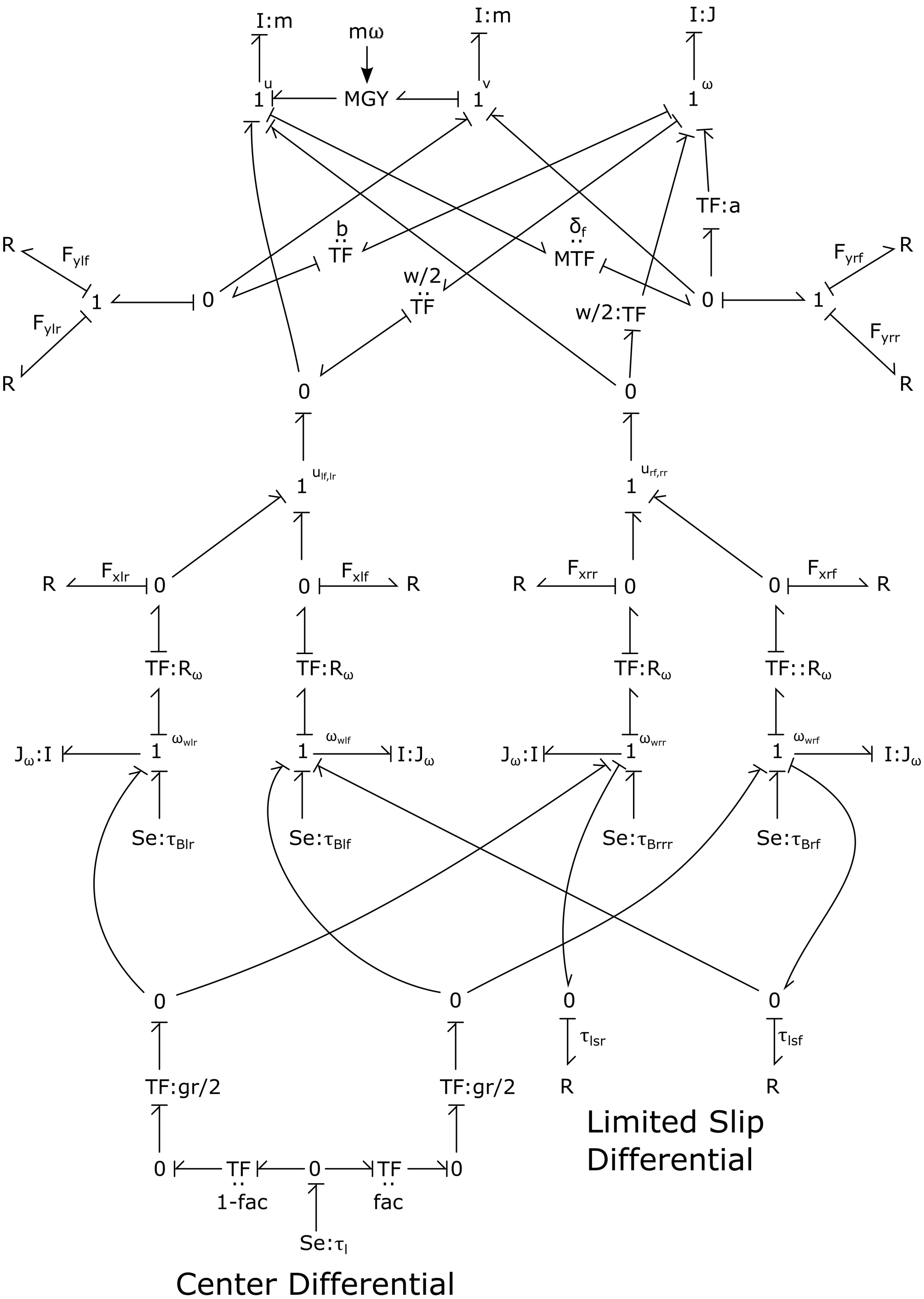

Figure 13: Bond Graph of Extended Bicycle Model

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