High-Order Implicit Shock Tracking

Tianci Huang, Charles Naudet, Matthew J. Zahr

University of Notre Dame
Aerospace and Mechanical Engineering
Email: mzahr@nd.edu

ABSTRACT

Shock tracking, as an alternative method to shock capturing, aims to generate a mesh such that element faces align with shock surfaces and other non-smooth features to perfectly represent them with the inter-element jumps in the solution basis, e.g., in the context of a finite volume or discontinuous Galerkin (DG) discretization. These methods have been shown to enable high-order approximation of high-speed flows and do not require extensive refinement in non-smooth regions because, once the non-smooth features are tracked by the mesh, the solution basis approximates the remaining smooth features.

In previous work [3, 4], we introduced an implicit shock tracking framework that re-casts the geometrically complex problem of generating a mesh that conforms to all discontinuity surfaces as a PDE-constrained optimization problem. The optimization problem seeks to determine the flow solution and nodal coordinates of the mesh that simultaneously minimize an error-based indicator function and satisfy the discrete flow equations. A DG discretization of the governing equations is used as the PDE constraint to equip the discretization with desirable properties: conservation, stability, and high-order approximation (both solution and geometry). By using high-order elements, curved meshes are obtained that track curved shock surfaces to high-order accuracy. The optimization problem is solved using a sequential quadratic programming method that simultaneously converges the mesh and DG solution, which is critical to avoid nonlinear stability issues that would come from computing a DG solution on an unconverged (non-aligned) mesh.

In this talk, we present a number of extensions to the implicit shock tracking method aimed at improving robustness for complex problems and extension of the framework to viscous and time-dependent flows [1, 2]. We use the proposed framework to solve a number of relevant shock-dominated flows in two and three dimensions, and demonstrate the potential of the method to provide accurate approximations to these difficult problems with local features on coarse, high-order meshes.

REFERENCES

[1] Tianci Huang and Matthew J. Zahr. A robust, high-order implicit shock tracking method for simulation of complex, high-speed flows. *Journal of Computational Physics*, 454:110981, April 2022.

[2] Andrew Shi, Per-Olof Persson, and Matthew J. Zahr. Implicit shock tracking for unsteady flows by the method of lines. *Journal of Computational Physics*, 454:110906, April 2022.

[3] Matthew J. Zahr and Per-Olof Persson. An optimization-based approach for high-order accurate discretization of conservation laws with discontinuous solutions. *Journal of Computational Physics*, 365:105–134, July 2018.

[4] Matthew J. Zahr, Andrew Shi, and Per-Olof Persson. Implicit shock tracking using an optimization-based high-order discontinuous Galerkin method. *Journal of Computational Physics*, 410:109385, June 2020.