Construction of High-Dimensional Quadrature Rules using Improved Node Elimination Algorithm

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Abstract

Numerical integration in two and higher dimensions is at the core of many numerical methods, such as Finite Element Method and Boundary Element Method. Whereas the theory of one-dimensional integration is well studied and understood, higher dimensions present significantly more challenges. In general, there is no universal algorithm to construct optimal quadrature rules, nor is it proven how many nodes are required to achieve theoretically optimal quadrature rules for a given degree, dimension, and domain of integration.

In the recent years, a node elimination proved to be a successful technique for computing efficient quadrature rules in two and three dimensions on domains such as cubes and tetrahedra(e.g. H. Xiao and Z. Gimbutas(2010)). However, to the best of author's knowledge, it had limited success in higher dimensions so far. In this work, we propose a new Node Elimination Algorithm, and other techniques to address limitations in four and higher dimensions without losing efficiency in two and three dimensions. One of the distinguishing features of our algorithm is a new Node Elimination technique, which performs linearization over a solution manifold, thereby generating accurate approximations to the quadrature rule while eliminating a node.

By using recursive schemes, we extend the Node Elimination algorithm to arbitrary degree and dimension for n-dimensional cubes, n-dimensional tetrahedra, and various polytopes.