A High-Order 3D Immersed Interface Method for Smooth Nonconvex Geometries

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Immersed Interface Methods (IIMs) are a class of Cartesian grid-based finite difference schemes that can achieve high order accuracy while allowing for complex non-conforming domain boundaries or physical interfaces. These schemes are particularly promising for simulations of interface-coupled multiphysics problems such as fluid-structure interaction and multiphase flow, in which maintaining an interface-conforming mesh can be prohibitively expensive. However, much of the existing work on IIMs has focused on 2D immersed bodies with concave geometry, avoiding several challenges that are unique to non-convex geometry and three-dimensional approximations.

In this presentation we demonstrate several improvements to existing IIMs that allow for high order PDE solutions in 3D domains with complex moving boundaries. In particular, we compare the utility of dimension-splitting and truly multidimensional polynomial reconstructions near immersed interfaces, and suggest a compromise approach that is robust to concave geometries and mindful of the computational overhead associated with moving interfaces. We also extend existing immersed interface boundary treatments for hyperbolic and parabolic processes to allow for the simulation of transport PDEs with nontrivial dynamics on both sides of an immersed interface.

These novel IIM discretizations are implemented in MURPHY, an open-source software framework for 3D multiresolution grids with wavelet-based adaptivity targeting massively-parallel computer architectures [1]. Through simulations performed within MURPHY, we assess the stability and convergence of our high-order 3D IIM when applied to standard transport PDEs. We also show that for representative 3D geometries the use of an IIM adds only a small computational overhead compared to a purely free-space finite difference scheme and does not negatively impact the weak scaling of our implementation. We conclude by presenting the results of large-scale simulations of interface-based PDE problems that illustrate the utility of our immersed method (see Figures 1 and 2).

References

[1] T. Gillis and W. M. van Rees. MURPHY–a scalable multiresolution framework for scientific computing on 3D block-structured collocated grids. *arXiv preprint arXiv:2112.07537*, 2021.

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