A high accuracy/resolution spectral element/Fourier-Galerkin method for the simulation of shoaling non-linear internal waves and turbulence in long domains with variable bathymetry

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A high-order hybrid continuous-Galerkin numerical method, designed for the simulation of non-linear, non-hydrostatic internal waves and turbulence in long computational domains with complex bathymetry, is presented. The spatial discretization in the non-periodic vertical and along-wave-propagation directions utilizes the nodal spectral element method. Such a high-order element-based discretization allows the highly accurate representation of complex domain geometry along with the flexibility of concentrating resolution in areas of interest. Under the assumption of the normal-to-isobath propagation of nonlinear internal waves, a third periodic direction is incorporated via a Fourier- Galerkin discretization.

The distinct non-hydrostatic nature of non-linear internal waves and, any instabilities and turbulence therein (*Lamb et al.* 2019), necessitate the numerically challenging solution of the well-known pressure Poisson problem subject to Neumann boundary conditions. As described in detail in *Diamantopoulos et al.* (2022), a defining feature of this work is the application of a domain decomposition approach, combined with block-Jacobi/deflation-based preconditioning to the pressure Poisson problem. Such a combined approach is particularly suitable for the long high aspect-ratio complex domains of interest and enables the efficient high-accuracy reproduction of the non-hydrostatic dynamics of non-linear internal waves.

A series of benchmarks of increasing complexity demonstrate the robustness of the flow solver. The series of benchmarks culminates into the application the computational tool at hand was actually designed for: the unprecedented turbulence-resolving three-dimensional simulation of a convectively breaking mode-one internal solitary wave (ISW) over a realistic South-China-Sea bathymetric transect and background current/stratification profiles. The turbulence-resolving inviscid simulations are motivated by the field observations of *Lien et al.* (2012 and 2014) and build on the lower-resolution 3-D study of *Rivera-Rosario et al.* (2022). Run on 6000 cores on NSF-XSEDE's Stampede-2 system, they use a resolution as low as 0.5*m* in all three directions in the breaking region and track an initially 1-*km*-long ISW over 80-*km* range of propagation from water depths of 900*m* to 300*m* depth. As the wave reaches a critical depth of approximately 450*m*, it breaks in a unique fashion: while the propagating wave continues to maintain its symmetric waveform and, therefore, its integrity, a plunging isopycnal from the rear of the wave produces a gravity current structure which propels itself through the wave interior (Fig. 1). A recirculating turbulent core is then established in the ISW interior capable of strongly mixing the water column and transporting particulate matter over long distances.

An overview of the physical insights, enabled by this new computational tool, first qualitatively discusses the differences in ISW convective breaking in two and three dimensions (Fig. 1). The critical role of turbulence in the latter case is considered, particularly in terms of generating shear instabilities in the rear of the wave. A quantitative discussion of marginal convective instability (*Chang et al.* 2021), turbulent kinetic energy spectra and turbulence energetics is then given.

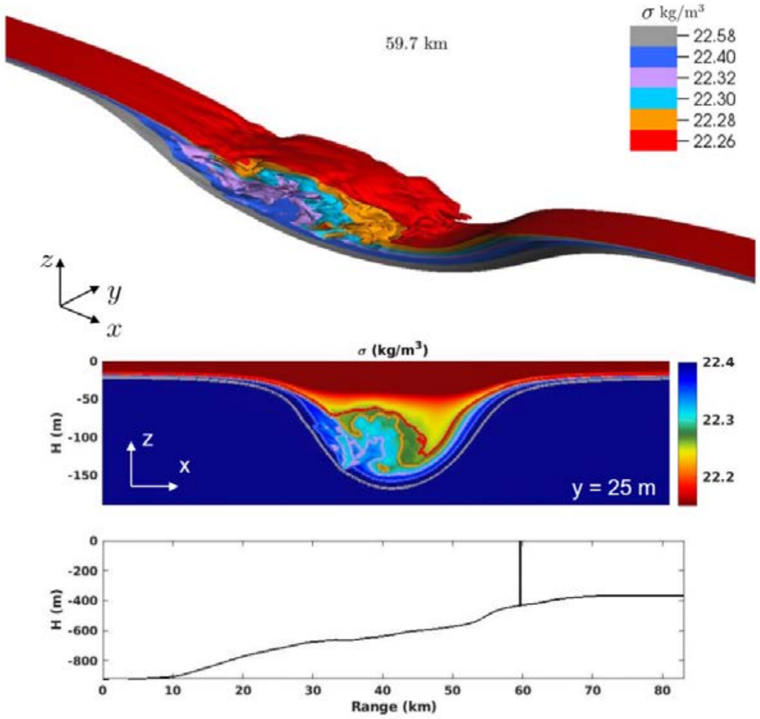

Fig. 1: Snapshot of Spectral-Element/Fourier-simulated 3-D simulation of an internal solitary wave during its propagation up the slope of the South China Sea. *Top panel*: Isosurfaces of potential density. *Middle panel:* isopycnal surfaces on *xz*-transect sampled through middle of transverse domain dimension. Note the gravity current feature that develops inside the wave and the well-mixed rear of the wave interior it produces. *Bottom panel*: bathymetry profile over full range of ISW propagation. The black line indicates the position of the wave trough at time sampled.

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