A fast direct solver for surface PDEs

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We introduce a fast direct solver for variable-coefficient elliptic partial differential equations on surfaces based on the hierarchical Poincaré–Steklov method. The method takes as input a high-order quadrilateral mesh of a surface and discretizes surface differential operators on each element using a high-order spectral collocation scheme. Elemental solution operators and Dirichlet-to-Neumann maps tangent to the surface are precomputed and merged in a pairwise fashion to yield a hierarchy of solution operators that may be applied in $\mathcal{O}(N \log N)$ operations for a mesh with N elements. The resulting fast direct solver may be used to accelerate implicit time-stepping schemes, as the precomputed operators can be reused for fast elliptic solves on surfaces. We apply the method to a range of scalar- and vector-valued problems on both smooth surfaces and surfaces with sharp corners and edges, including the static Laplace–Beltrami problem, the Hodge decomposition of a tangential vector field, and some time-dependent reaction–diffusion systems.