A comparison of multiwavelets and machine learning troubled cell detection in modal discontinuous Galerkin methods

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Spurious oscillations, such as Gibbs phenomenon, are artifacts that occur in higher-order numerical methods that create non-physical effects, e.g. negative density or pressure. These oscillations need to be accounted for and eliminated to maintain physical relevance and accuracy in the approximations. In order to apply appropriate numerical adjustments using slope limiting or other techniques, such spikes or discontinuities would need to be correctly identified. Traditional troubled cell indicators include minmod- and modified minmod-based limiters that result in falsely detecting smooth extrema as troubled cells and reducing polynomial order. Recently, artificial neural networks (ANNs) have been shown as troubled cell indicators in discontinuous Galerkin (DG) methods and hybrid-compact WENO finite difference schemes [1, 2] . However, these machine learning methods have not been thoroughly compared to multiresolution methods such as multiwavelets, which inherently capture discontinuity data across scales without the need of high computational cost for machine learning training.

In this work, we present a comparison study of multiwavelets and machine learning in troubled cell detection in a higher-order, modal DG semi-discrete variational scheme for hyperbolic systems. By extracting the fine details through multi-resolution analysis in the multiwavelet approach, we can analyze the global information in the domain and apply theoretical thresholding and outlier detection to identify cells that are troubled. Additionally, we have trained classifiers on smooth and discontinuous data, enabling a machine learning solution to discontinuity detection. In the talk, the multiwavelets and machine learning approaches will be compared in their construction, framework, and computational cost. Discontinuity detection results for the two approaches are presented for linear, Burgers, and Euler problems.

References

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