A MODEL FOR USE IN THE PROGNOSIS OF TENDINOPATHY

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ABSTRACT

This work describes the viscoelastic behavior of a tendon. In this paper we develop a mathematical model of the Posterior Tibial Tendon (PTT) for assisting in the prognosis of tendinopathy. The mathematical model is derived from a generalized Maxwell model for viscoelastic materials. To validate the model, a strain is asserted and the associated stress represented. Using the simulated response to a given stress, the mathematical model of the PTT is validated for a response indicative of a healthy tendon.

Keywords: Posterior Tibial Tendon, tendinopathy prognosis, mathematical model, simulation.

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1 INTRODUCTION

The Posterior Tibial Tendon (PTT) is comprised of several layers of viscoelastic structures and extends down the back of the leg then along the bottom of the foot. PTT dysfunction (PTTD) is a condition which affects the elasticity of the PTT.

Long term effects of PTTD can result in a condition commonly known referred to as “Flat Foot”. Symptoms of (PTTD), or tendinopathy of the PTT, include moderate to severe pain on the inside of the foot which can radiate along the length of the PTT. This injury may result from chronic overstressing of the PTT and may result from degeneration of the PTT due to aging. Given this is a painful condition, it has significant impact on the quality of life for those affected (Gao et al., 2016).

Those diagnosed with PTT dysfunction often have therapy or surgical correction recommended as alternatives. While there is a high degree of success in the restoration of function and relief of the symptomatic pain for many patients recommended for therapy (Alvarez et. al., 2006), there are many who will ultimately be recommended for surgery. The recommendation for therapy or surgery is qualitative and it is often unknown whether a recommendation for therapy is too conservative or a recommendation for surgery is premature.

By providing a mathematical model of the PTT which can be used in conjunction with non-evasive ultrasonic measurement to assist in the prognosis of PTTD. This is intended to provide the practitioner an additional quantitative assessment to assist in the prognosis of the severity of the PTTD.

2 MODEL DEVELOPMENT

There is a generalization of the Maxwell Model for viscoelastic material which neatly accounts for the contributions at the various levels of the tendon structure to the over strain response for a given stress. This work models the behavior of the PTT as a viscoelastic material using a Generalized Maxwell model to mimic the phased interactions of the structural layers of the PTT.

2.1 Tendon Structure

For the purpose of this work, the PTT is modeled as being comprised of several layers of viscoelastic material. In this model of the tendon, there are multiple elements contributing to the tendon response in seven major layers. The tendon is comprised of a number of fascicles, which are comprised of a large number of fibers, the outer surface of which is arranged in a wave-like pattern. The fibers are composed of again an large number of fibrils, the skin of which has an wave-like pattern which is approximately orthogonal to that of the fibers.

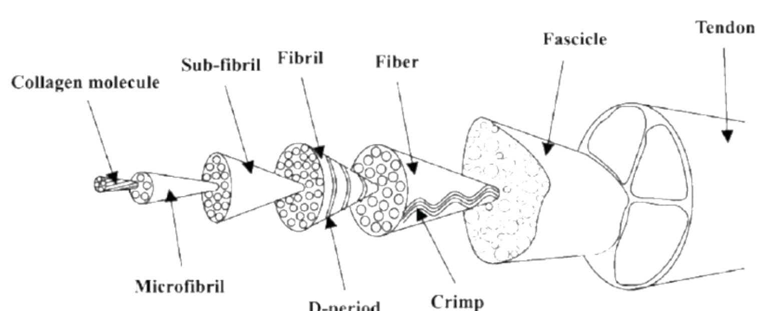

Figure 1: Hierarchical structure of a tendon (Adapted from Kastelic et al. (1978)).

The fibril is comprised of many sub-fibrils which are themselves comprised of many micro-fibrils. Finally, the micro-fibrils are made up of strands collagen molecules. Each of these layers are contained in an encompassing material, and have the common property of being bundled at each level with like structures to comprise the next higher strand. In the end, having the effect of the tendon being bundles of bundles of strands seven layers deep. This structure is shown in Figure 1 (Kastelic et al, 1978).

2.2 Stress, Strain

Stress in this work is regarded as the force per unit area, $\sigma = F/A$. Strain, then, is regarded as the ratio of the change in the length under a given stress to the original length of the specimen, $\epsilon = \Delta l/l_0$. The relationship between stress and strain is given by Hooke's law where stress is the product of the Young's Modulus and the strain, $\sigma = E\epsilon$.

An ideal viscous material will react to stress linearly over time resulting in a permanent deformation of the material upon removal of the stress, refer to Figure 2 (Pekaje, 2016) for the schematic and response curves . An ideal elastic material will deform instantaneously when a stress is applied and return to the original shape immediately and fully upon the removal of the stress, refer to Figure 3 (Pekaje, 2016).

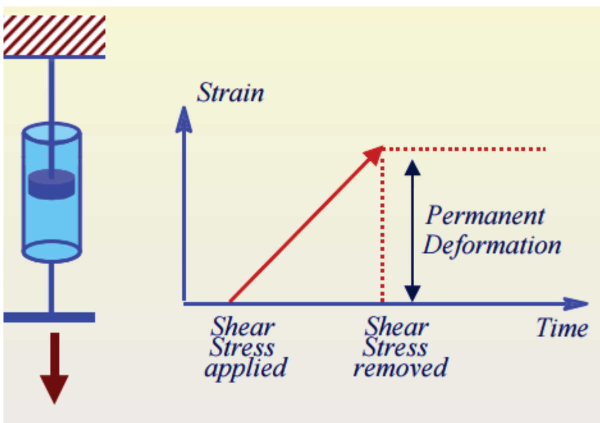

Figure 2: Viscus Schematic and Response.

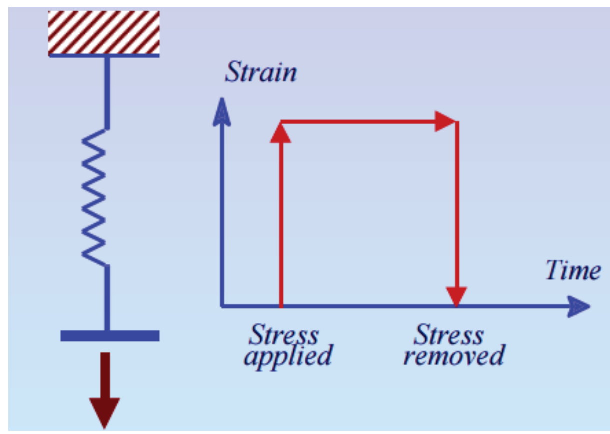

Figure 3: Elastic Schematic and Response.

2.3 Maxwell Model Viscoelastic Model

An ideal viscoelastic material, as envisioned by Maxwell, will react nonlinearly over time to the application of stress and with the removal of the stress has an inelastic response, though the original shape will never be fully restored. Refer to for the schematic and response of an idealized viscoelastic material in the Maxwell Model shown in Figure 4 and in Figure 5, respectively.

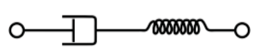

Figure 4: Schematic of Maxwell Model for Viscoelastic Material (Pekaje, 2016).

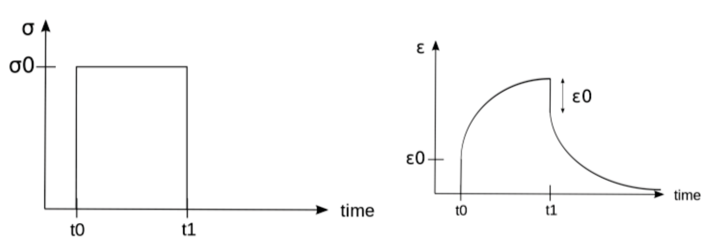

Figure 5: Viscoelastic Material stress and strain response (Pekaje, 2016).

In the development of the Maxwell Model for viscoelastic materials, Maxwell started with Hooke's Law and generalized that to allow the stress to vary over time. Maxwell further asserted that the strain rate is the sum of the viscus material response over time and the elastic response over time.

$$\sigma = E\epsilon$$

$$\frac{d\sigma}{dt} = E\frac{d\epsilon}{dt} \Rightarrow \frac{d\epsilon}{dt} = \frac{1}{E}\frac{d\sigma}{dt} \quad (elastic\ response)$$

$$\sigma = \eta\frac{d\epsilon}{dt} \Rightarrow \frac{d\epsilon}{dt} = \frac{\sigma}{\eta}(viscous\ response)$$

$$summing, \qquad \frac{d\epsilon}{dt} = \frac{\sigma}{\eta} + \frac{1}{E}\frac{d\sigma}{dt} \quad (Maxwell's\ Model\ for\ viscoelastic\ response)$$

As noted previously, Maxwell's Model for viscoelastic response does not allow for complete restoration of the material to its original shape. Further, the Maxwell Model is sufficient for simple, single element system, of which the PTT is not an example. To begin to accommodate the actual complexities of the PTT, this work employs the Generalized Maxwell Model.

2.4 The Generalized Maxwell Model

The Generalized Maxwell Model (GMM) begins to account for the complexities of an actual PTT by providing for several factors. The GMM adds a single, ideal elastic element which will allow for the entire structure to return to the original shape over time. This single ideal elastic element can account for either the natural tendency for a healthy tendon to return to its original shape or may be used to account for healing over an extended period of a tendon damaged due to overstress. As shown in the following equation and represented in Figure 6, the GMM provides an unlimited number of individual viscoelastic elements.

$$\frac{d\epsilon}{dt} = \mathrm{K}_e + \sum_{n=0}^{\infty} \tau_n + \kappa_n \frac{d\sigma}{dt}$$

where $\tau_n = \frac{\sigma}{\eta}$ and $\kappa_n = \frac{1}{E}$ from Maxwell's Model for viscoelastic response above.

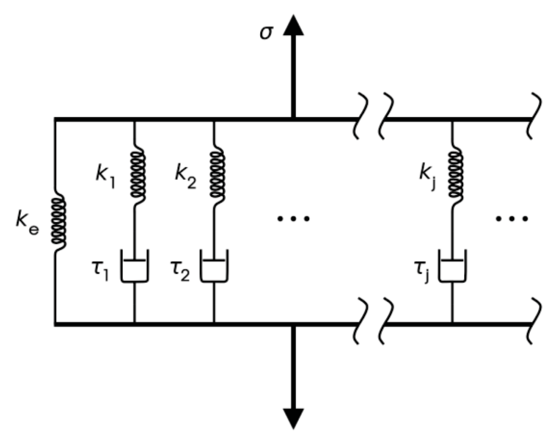

Figure 6: Schematic of Maxwell-Wiechert Model (Pekaje, 2016).

These elements can be used to refine the overall model to represent the actual elemental strands of the complete tendon. Such elements in the GMM each have their own constants of viscosity and elasticity, τ_n and κ_n, which may be interpreted to represent the differing response times of the strands, or bundles of strands, which they represent. In this way, the individual response times of the various levels of the structure of the tendon are represented. These constants can be adjusted to model the behavior of entire groups of strands at the various levels of the tendon structure. For example, the tendon can be represented with a single viscoelastic element in parallel with an ideal elastic element – though this level of resolution may be insufficient for use in practice.

$$\frac{d\epsilon}{dt} = \mathrm{K}_e + \tau_0 + \kappa_0 \frac{d\sigma}{dt}$$

A slightly more practical representation of the tendon is to have one element of the GMM representing all of the collagen strands in the micro-fibril, one representing all of the micro-fibril strands in the sub-fibril, etc. until there is a final element which represents the complete tendon, where the constants τ_n and κ_n have unique values for each group or bundle. In this case, the equation representing the system is as follows and the constants of elasticity and viscosity, $\tau_{0,\dots,6}$ and $\kappa_{0,\dots,6}$., each represent the total elastic and viscous response of the tendon.

$$\frac{d\epsilon}{dt} = \mathrm{K}_e + \sum_{n=0}^{6} \tau_n + \kappa_\mathrm{n} \frac{d\sigma}{dt}$$

In this way, the GMM can be refined to model each individual strand for each group of the tendon structure by incorporating an appropriately large number viscoelastic elements.

2.5 Non-Linearity of the PTT

The response of a healthy tendon to a stress is known to be nonlinear (Massoud, 2013). At this stage in the development of the model, the known non-linearity of the response remains to be addressed. As stress is increased from 0 to approximately 10 MPa, strain increases rapidly to more than 1/5 of the maximum strain before permanent damage is incurred. It is during this range that the wave-like structures of the fibers and fibrils flatten out. As the wave-like structures flatten, the tendon enters a linear region where the strain is increased linearly with the increase in the stress. In the neighborhood of 75 MPa and ¾ of the maximal strain, the tendon begins to experience partial failure. Around 100 MPa, total failure of the tendon is imminent. This occurs when strain reaches around 8%. Refer to Figure 7 for a graphic representation of preceding description (Massoud, 2013).

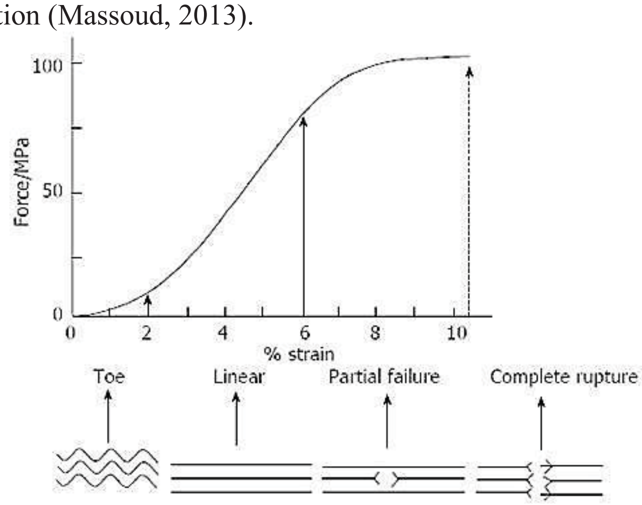

Figure 7: Non-linear tendon response to induced stress (Massoud, 2013).

To model this known non-linearity, the constants of elasticity and viscosity are expended as exponential sums.

$$\frac{d\epsilon}{dt} = \mathrm{K}_e + \sum_{n=0}^{\infty} \tau_n + \kappa_\mathrm{n} \frac{d\sigma}{dt}$$

$$but\ \tau_n = \frac{\sigma}{\eta}\ and\ \kappa_n = \frac{1}{E}, so,$$

$$\frac{d\epsilon}{dt} = \mathrm{K}_e + \sum_{n=0}^{\infty} \frac{\sigma}{\eta_n} + \frac{1}{E_n}\frac{d\sigma}{dt},$$

where, $\eta_n\ and\ E_n$ are the constants of viscosity and elasticity, respectively. Noting that the constant of elasticity is the Young's Modulus, there are many ways in which the above equation can be extended to account for the non-linearity in the toe and failure regions of the stress/strain curve. This work extends the constant of viscosity to include the rapidly increasing strain in the failure region. Further, this work extends the constant of elasticity to include the rapidly increasing strain in the toe region. The development of these extensions follows.

2.5.1 Extension to the constant of viscosity

The constant of viscosity is extended to include the rapidly increasing strain in the failure region of the tendon with the rationale that when the tendon is stressed into this region, the structure becomes plastic with continued deformation resulting in a reluctance (or inability) of the structure to return to the original shape. As such, and upon failure, the tendon approaches an ideal viscous material.

To extend the constant of viscosity to represent the non-linear failure region, this work constructs an exponential function of σ, the induced stress, which sufficiently increases the coefficient of viscosity in the failure region while contributing very little to the toe and linear regions.

$$\eta_n \rightarrow \eta_n(\sigma) = \eta_n + e^{C\sigma}$$

where C is a modifiable constant such that $e^{C\sigma} \rightarrow 0$ as $\sigma \leftarrow 80$MPa, and if the reader will pardon the dual use of η_n

2.5.2 Extension to the constant of elasticity

The constant of elasticity is extended to include the rapidly increasing strain in the toe region of the tendon with the rationale that when the tendon is stressed in this region, the tendon remains most likely to return to the original shape. As such the tendon approaches an ideal elastic material.

To extend the constant of elasticity to represent the non-linear toe region, this work constructs an exponential function of σ, the induced stress, which sufficiently increases the coefficient of elasticity in the toe region while contributing very little to the failure and linear regions.

$$E_n \rightarrow E_n(\sigma) = E_n + e^{C\sigma}$$

where C is a modifiable constant such that $e^{C\sigma} \rightarrow 0$ as $\sigma \rightarrow 10$MPa, and if the reader will pardon the dual use of E_n

2.6 Completing the model

In this way, the complete model of the PTT for this work represents the individually timed responses of the various structures of the tendon in as fine a detail as may be desired by the practitioner as to provide a sufficiently quantitative prediction for the elasticity of the PTT as an assessment of the health thereof as may be necessary or desired by the practitioner. The following equation represents the final model, where m represents the number of Maxwell elements used to represent the viscoelastic elements of the tendon.

$$\frac{d\epsilon}{dt} = \mathrm{K}_e + \sum_{n=0}^{m} \frac{\sigma}{\eta_n + e^{C\sigma}} + \frac{1}{E_n + e^{C\sigma}}\frac{d\sigma}{dt}$$

3 SIMULATION OF THE MODEL

3.1 Simulation Results

The model is exercised in a single dimension to show that the strain associated with an induced stress is consistent with (Massoud, 2013). Limiting the strain to approximately 6% with an appropriately parameterized model shows this for the single dimension over the first second of the induced stress.

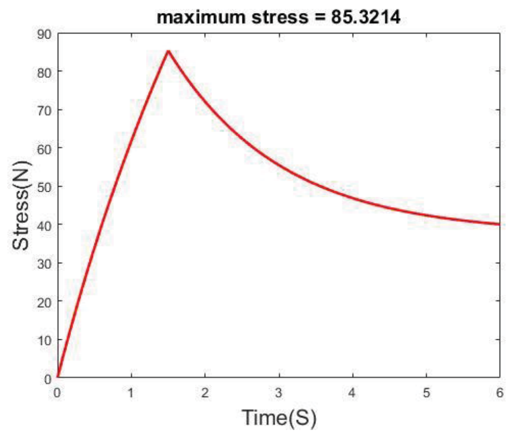

Figure 8: Induced Stress over Time with Strain limited to ~6%.

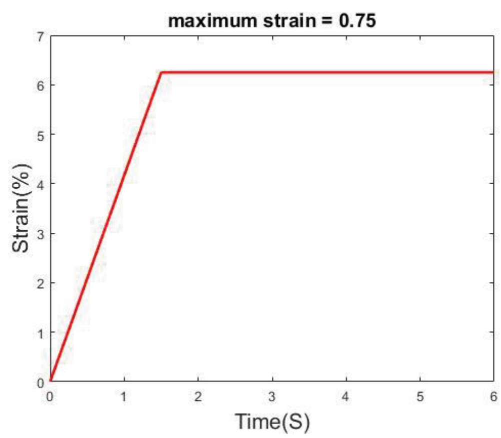

Figure 9: ~6% Strain over Time to produce stress in linear region.

This representation uses m=3 elements to represent the major elements of the tendon and is parameterized with eta_1 = 1160, eta_2=14230, eta_3=153160, K_e= 50 and E_n=1/3.

4 DISCUSSION

We have developed a mathematical model which represents the strain of a viscoelastic material when excited with a known stress. We have shown that that model results in motion consistent with that of an actual tendon in the application of a particular stress over a one second period.

5 KNOWN OMISSIONS AND FUTURE WORK

There remains to establish the representation of the model against actual ultrasound data from a known healthy tendon for use in assisting in prognosis of PTTD. The automation of segmentation the motion of the tendon from actual ultrasonic data will be performed in a later paper. From the automated segmentation of the ultrasonic data, the model can be used to predict the expected behavior of a healthy tendon.

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